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Mathematics 8

Learn EveryWare

Unit 4

Polygons, Prisms, and Cylinders

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Mathematics 8

Learn EveryWare

Unit 4

Polygons, Prisms, and Cylinders

Mathematics 8
Unit 4: Polygons, Prisms, and Cylinders
Student Module Booklet
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Unit 4: Polygons, Prisms, and Cylinders

Unit 4 Introduction

At a grocery store, food products are contained in a variety of shapes. Some packages are floppy—like a sack of potatoes or a bag of oranges. Other packages are firm and have defined edges. Think of a box of cereal or a tin can of soup. Generally, boxes of chocolates that have flat surfaces are examples of shapes known in geometry as prisms. Tin cans that have a curved side and circular ends have a shape known as a cylinder. These packages are all examples of three-dimensional (3-D) objects—objects that have not only length and width but also depth (or height).

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In Unit 4 you will study prisms, cylinders, and other 3-D objects. You will lear
n to represent 3-D objects as flat shapes that can be folded back into their related 3-D shapes. You will also develop and apply strategies to determine the following:

- the total area of the surface of a prism or cylinder
- the value representing the space an object takes up

As you progress through this unit, you will gain the knowledge of 3-D objects and associated skills you need to design a storage container and a display cabinet.

This unit will help you answer the following critical question: How are prism and cylinder shapes used in the storage and display of products?

The new concepts and skills related to 3-D objects will be presented in eight lessons.

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Unit 4: Polygons, Prisms, and Cylinders

Lesson 1: Views of Three-Dimensional Objects

Lesson 2: Nets of Three-Dimensional Objects

Lesson 3: Surface Area of a Prism

Lesson 4: Surface Area of a Cylinder

Lesson 5: Understanding Volume

Lesson 6: Volume of a Prism

Lesson 7: Volume of a Cylinder

Lesson 8: Solving Problems Involving Prisms and Cylinders

In this unit you will complete various types of assignments:

- interacting with other students through a discussion board
- adding samples of your work to your Math 8 folder
- completing sets of questions for each lesson
- completing a unit problem at the end of the unit
- possibly writing a unit test

Be sure to submit your assignments according to the directions that will be provided throughout this unit.

Strategies for Success

In order to support your success in this unit, follow these strategies.

Strategy 1

Make two of the foldable study tools according to the detailed instructions on pages 162 and 244 of your textbook. Although these activities may not be graded, you will benefit from these tools. Keep the points you record in mind as you develop and use these study tools:

- As you are working through the lessons, add formulas, examples, and vocabulary words according to instructions in the Connect section of each lesson.

- Each foldable can serve as a quick reference guide and will help you save time when you are ready to study for a unit test or work on your unit problem.

Strategy 2

In this unit you will be looking at Chapters 5 and 7 in your textbook. Chapter 5 spans pages 160 to 193 of your textbook and Chapter 7 is found on pages 242 to 281.

- Take time to flip through the pages of each of these chapters in your textbook.

- View the introductions to the two textbook chapters, and make a comparison of what each chapter is about. What 3-D objects can you recognize?

- Look at all illustrations, margin features, and main titles to get a sense of what you will learn about.

Unit 4 Problem

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This unit ends with a Unit Problem to complete. For this unit problem, you are to design a container to be sold in a specialty store. The owner of the store indicates the container must be appealing, practical, and decoratively labelled—not like the everyday containers you may be familiar with. You are also to design a display case for the manufactured containers and an ad to promote the sale of the containers.

The Unit Problem has two parts.

- Part 1 should be completed along with Lessons 1, 2, 3, and 4.
- Part 2 should be completed as you do Lessons 5, 6, 7, and 8.

This schedule will allow you to apply the knowledge and skills of 3-D objects as you learn them. Go to the Unit Summary to find out more about the Unit Problem.

In Grade 7 you studied characteristics of two-dimensional (2-D) figures. You determined the circumference of circles and the area of triangles, rectangles, and circles. These figures are all two-dimensional, but what you learned about them will help you study the characteristics of three-dimensional objects. That's because the surfaces of 3-D objects—think of prisms and cylinders—are made up of familiar 2-D shapes you studied before. You should now be ready to begin the lessons of this unit.

Unit 4: Polygons, Prisms, and Cylinders

Lesson 1: Views of Three-Dimensional Objects

Get Focused

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Do you ever help out with the grocery shopping? If so, then you've see that many identically shaped packages are stacked on shelves. Stacking allows the shelves to display a lot of merchandise to the shopper.

Imagine the kinds of three-dimensional (3-D) objects you could build by stacking a few cube-shaped blocks. You could make a tall tower, a low platform, or even a small staircase. Once you build your structure, you may want to describe to a friend what your structure looks like. That could be a little tricky, because what your structure looks like depends on how you look at it.

Read

Turn to page 164 in your textbook *MathLinks 8*. Sable and Josh are supposed to be making the same 3-D object without looking at each other's work. They would have to be able to describe what their objects look like from different positions or angles. Looking from different positions or angles gives a different view of the same object.

You may need double the time you have used for other lessons. Here, you will meet new concepts relating to visual images. Your learning will come through hands-on activities in which you build numerous objects.

In this lesson you will draw the top, front, and side views of some 3-D objects. You will also build 3-D objects based on what the top, front, and side views are like. You will also review transformations of 2-D objects.

This lesson will help you answer the following critical questions:

- How do you draw a 3-D object on a flat surface?
- Why is the appearance of an object important to a merchant?

You need ten unit blocks for this lesson. You can use wooden or plastic building blocks, alphabet blocks, or even sugar cubes. Just make sure the blocks are cube-shaped and easy to handle. You will also need Isometric Dot Paper.

Assignments

Your assignment will consist of the following:

- posting and responding to your discussion board
- adding to your Math 8 folder
- completing Unit 4: Lesson 1 Question Set

Special words can make describing an object to someone else easier to do.

Study "Literacy Link" on page 164 of your textbook. See how **face**, **edge**, and **vertex** refer to different parts of a 3-D object. Go to the Math 8 Multimedia DVD, and click on "Face," "Edge," and "Vertex" for more information.

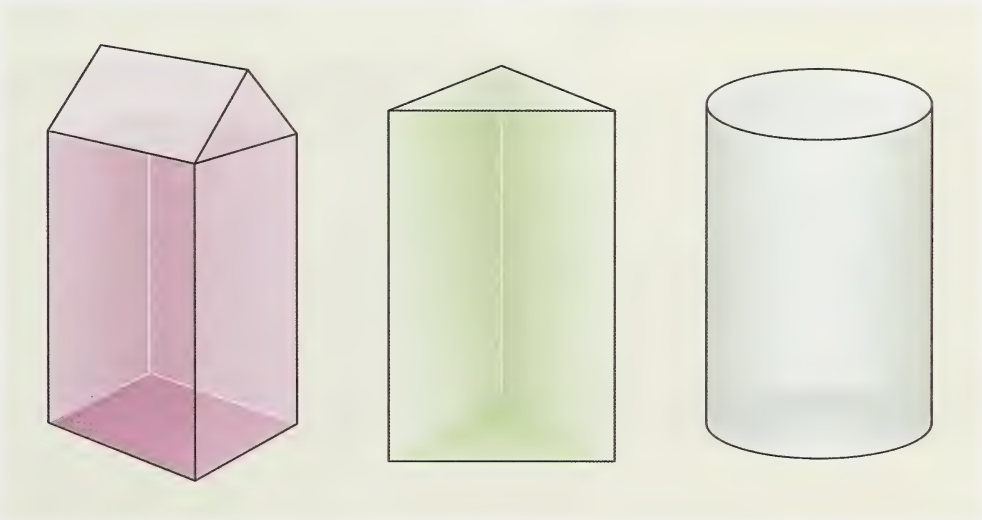
face: a flat or curved surface

edge: a line segment along which two surfaces meet

vertex: a point where three or more edges meet

Try This

TT 1. Look at the following objects. Describe one of the objects to your partner by indicating its features. Use the words *vertex, face*, and *edge* in your description.



Left to right, the objects are Figure A, Figure B, and Figure C.

Note: The challenge is to describe one of these objects to someone else using the words *vertex, face*, and *edge.* You may not use the words *cylinder* or *prism.* This task will make you pay attention to the features of the object and not to some group the object belongs to.

You will need a partner. Your partner may be someone you know from your discussion board. Agree to use instant messaging in responding to the question. Your discussion may be like Sable and Josh's talk on page 164 of your textbook *MathLinks 8*. You will know that your description is effective if your partner can work out the object you are describing.

Discuss and Share

Ask your partner to guess if you described Figure A, Figure B, or Figure C. Then read your partner's description, and try to guess which figure your partner chose. Also, let each other know how the descriptions could be improved.

Communicate by instant messaging and respond to each other's ideas. That is, work together. Do not just list ideas independently of each other. Send your partner a description that you agree is complete and error-free. Save your transcript from your instant messaging and place it in your Math 8 folder.

Explore

In TT 1, you discovered how important it is to have common language to describe the shape of three-dimensional objects. This vocabulary is very important when you are describing objects, because it allows an accurate and clear understanding about what the object looks like to be communicated. Now that you have some common vocabulary to describe objects, see how you can describe 3-D objects that you build.

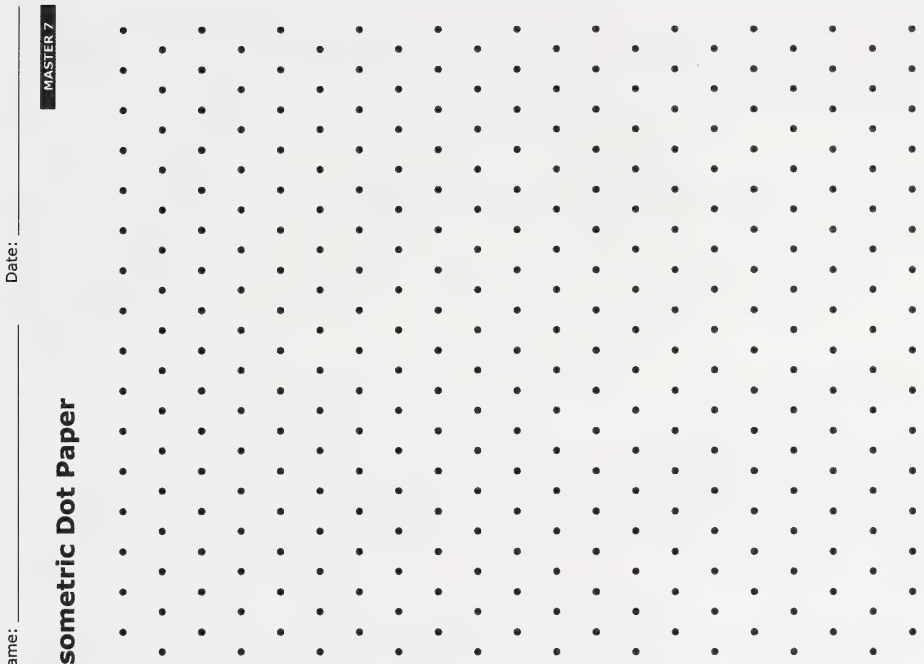
In "Explore the Math" presented on pages 164 and 165 of your textbook, you will build a 3-D object and then describe the object to a partner. Your partner may be assigned by your teacher, or your partner may be someone you know from the discussion board.

You and your partner will each need ten unit blocks for this lesson. If you prefer, you can use wooden or plastic building blocks, alphabet blocks, or even sugar cubes instead. You may go to the Math 8 Multimedia DVD, and find "Isometric Dot Paper." You may print out Isometric Dot Paper when you need it to draw your object. Position the paper sideways, and then draw on it.

MASTER 7

Name: _____ Date: _____

Isometric Dot Paper



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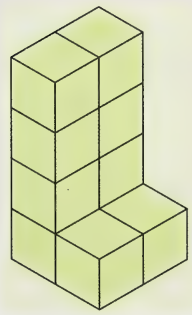
Try This

TT 2. Complete "Explore the Math" questions 1, 2, and 3 on page 164. Discuss and share your answers with your partner. You know your descriptions are effective when your partner can build the same object as you.

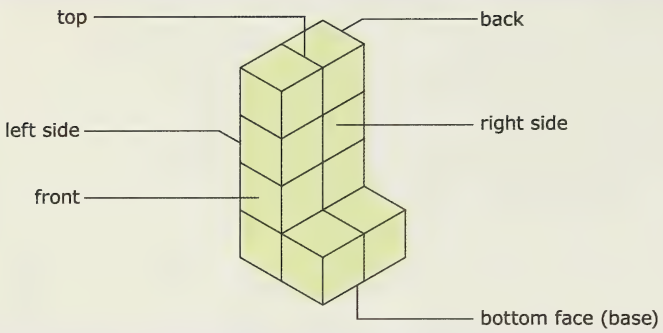
My Guide

Here's a sample response to "Explore the Math" questions 1, 2, 3, and 4. (You are to do question 4 in TT 4.)

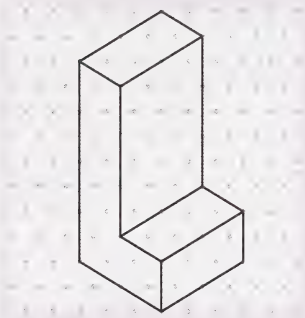


At first, you could describe the front face as having the shape of a 3-D letter L. The letter is made of two layers of blocks. You could continue by mentioning the height of the letter as well as the length of the horizontal part of the L.

The L is made of two layers of blocks. Its base is a square made of four blocks.



Once you label the faces, your description can start this way: The base looks like a square made of four blocks. The front looks like an L, and the back is the same shape as the front (but in reverse). The top is made of two levels. Each level consists of two blocks side by side.



You should now have a good idea of what do in your Explore.

Place a copy of your answers in your Math 8 course folder.

TT 3.a. Draw a picture of the object you made.

b. Draw a picture of the other object you had made to be identical to your partner's.

Discuss and Share

If you have a digital camera available, take a picture of each of your objects. Share your drawings and your pictures with your partner.

Then discuss and share your answers with your partner. Pay attention to any differences between the drawings of objects that were supposed to be identical. Discuss any reasons that might account for the differences. Discuss this question: What could have been communicated differently to ensure that the objects created were the same?

TT 4. Complete "Explore the Math" question 4 on page 165.

Place a copy of your answers in your Math 8 course folder.

TT 5. Complete "Explore the Math" question 5 on page 165. Your conclusions may differ from others based on what objects you made and described.

Discuss and Share

Discuss and share with your partner the statements you make to back up your conclusions. Correct each others' statements. Use feedback from your partner to arrive at your final conclusions.

Place a copy of your final answers in your Math 8 course folder.

Connect

Let's now build on what you and your partner discovered during the Explore.

Read

When you drew your 3-D object on isometric dot paper, the 3-D object was displayed on an angle so that three faces were visible. How many faces are visible when you draw the top, front, or side view of an object?

Read "Example 1: Draw and Label Top, Front, and Side Views" on page 165 of your textbook to see the various drawings of two familiar objects. Can you see that, for one of the objects, the three views are close to being the same shape and area? The other object has three views that vary a great deal in shape and size. You can see that only one face is visible in each view.

Self-Check

SC 1. Complete "Show You Know" at the top of page 166 of your textbook to draw your own views of an object. You may find that Square Dot Paper will help you make the drawings more easily. On this paper, you can easily identify horizontal and vertical rows of dots. As you know, slanting lines on isometric dot paper seem to jump out at you more.

Compare your answers in the Appendix.

No single one of the views of a 3-D object that you just drew represents the 3-D object very well. It takes some practice to get your mind wrapped around views. The next example will help you interpret views.

Read

"Example 2: Sketch a Three-Dimensional Object When Given Views" on page 166 of your textbook shows three views of the same object. Notice that the top view gives no indication of the stair-case pattern of the object. Neither does the side view. It's only the front view that indicates this pattern. You really need all three views to get the complete picture.

Self-Check

SC 2. Now here's an interesting task! Complete "Show You Know" at the bottom of page 166 of your textbook. You'll draw a 3-D object from just its top, front, and side views on isometric dot paper. A drawing on this paper will help you visualize the object in its three dimensions.

My Guide

You may want to build a model of what you think the 3-D object is by using unit cubes and possibly some adhesive to keep any unsupported blocks in place. Then look at your model from the top, front, and side. If your views are different from the given views, then you know you have to adapt your 3-D object.

Compare your answers in the Appendix.

Watch and Listen

Open "Drawing Dimensions" on the Math 8 Multimedia DVD to gain more insight into how the views of a 3-D object relate to the object itself. Just follow the directions. Challenge yourself with the quiz questions, which check your understanding.

Note that in this learning resource, the left view is the view from the back of the object.

Read

You have seen the views of an object labelled as top, front, and side views. But determining which view is from what angle depends on the **orientation** of the object.

Go to "Rotation of a Cube" on the Math 8 Multimedia DVD to see how an object's orientation can change.

orientation: the placement of an object so it faces in a particular direction; the angular direction of an object

The orientation of an object changes when the object is rotated.

Read "Example 3: Predict and Draw the Top, Front, and Side Views After a Rotation" on page 167 of your textbook to see how rotation of an object causes the top, front, and side views to change.

My Guide

Do you find it hard to see which face is which for the computer tower? Get a used cardboard box. Sketch the different faces on the box as you'd see them on the computer tower. Then rotate the box 90° on its base. See which faces are visible from the top, front, and side views.

Self-Check

SC 3. Complete the question from "Show You Know" at the bottom of page 167 of your textbook. Draw the views with some features such as lettering, page edges, and the extension of the cover past the pages.

My Guide

Having trouble picturing the rotation of the textbook about its spine? These photographs should help you with the before and after:

Bruce McAskill et al., *MathLinks 8* (Toronto: McGraw-Hill Ryerson, 2008). Reproduced by permission.

Compare your answers in the Appendix.

In the previous exercise you observed that when 3-D objects were rotated, the views also changed. Of course, the 3-D object stayed the same shape after the rotation.

Recall that in Grade 7 mathematics you explored the rotation of polygons. Rotation was considered a transformation. A transformation is a change that makes a polygon go to a new position or take on a new orientation. The other transformations you explored in your previous mathematics course were translation and reflection. None of these three transformations changes the size or shape of the polygon. That is, a polygon that has come about by rotation, translation, or reflection is **congruent** with the original polygon. For more information, look up "Rotation," "Translation," and "Reflection" on the multimedia DVD.

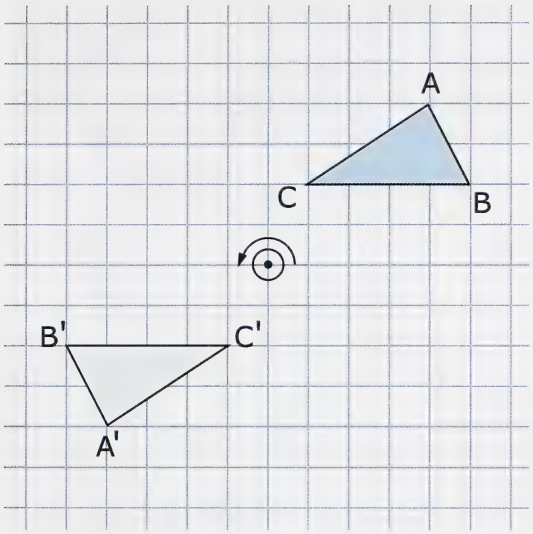
Joe Sohm/Photodisc/Getty Images

congruent: having the same size and shape

In the study of geometrical figures, a rotation is a transformation in which a polygon is turned around a fixed point. A rotation is described by angle and direction. The size of the angle can be described in two ways—as a fraction of a turn or in degrees of an angle. (You can look up "Degrees of an Angle" on the Math 8 Multimedia DVD.)

Example: Describe a Rotation

Suppose ΔABC is rotated so that it goes to the position of ΔA′B′C′. ΔA′B′C′ is called the image of the rotation.

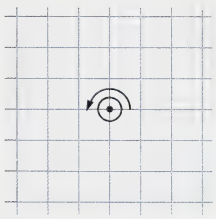


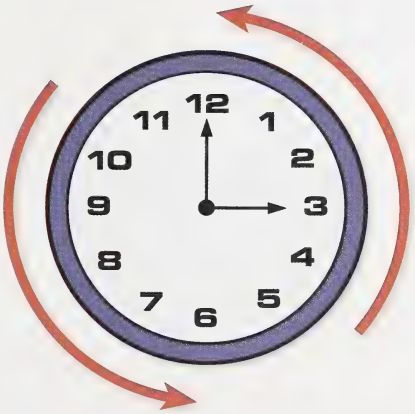
Read the accent mark (′) that is above and to the right of each letter as *prime*.

Describe the rotation of the triangle.

Solution

The turn arrow represents rotation of one-half a turn (or 180°) in the counterclockwise direction.



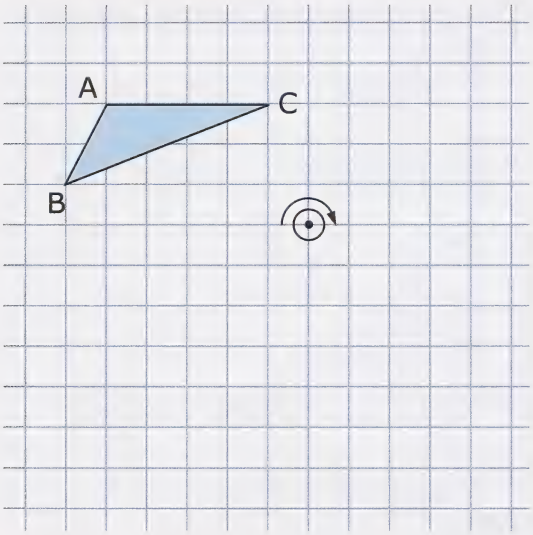


Check that the image has been placed according to this rotation. You do this by checking the following:

- The line segments from the turn centre to corresponding vertices are the same length. They differ only in their position. For example, see that the line segment from the turn centre to A is the same length as the line segment from the turn centre to A′.
- The line segment to the image vertex is one-half a rotation from the line segment to the corresponding vertex in the original figure. For example, see that the line segment from the turn centre to A′ is one-half rotation from the line segment from the turn centre to A.

So, the rotation is one-half turn counter clockwise from the original figure to its image.

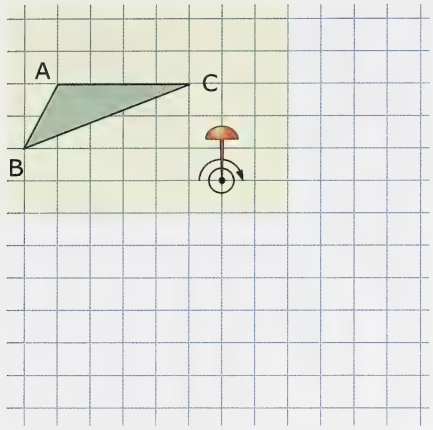
Example: Find the Image of a Rotation



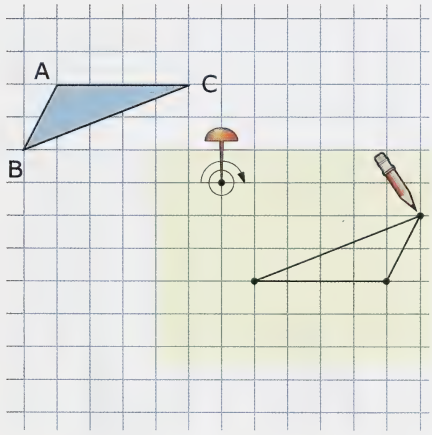
Solution

Step 1: Note the direction and the size of the rotation. The rotation is one-half a clockwise turn or 180° clockwise.

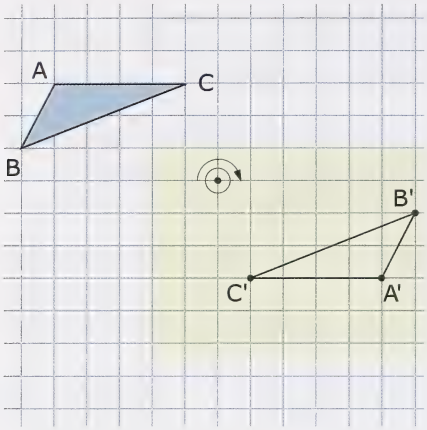
Step 2: Cover the figure and rotation centre with tracing paper. Push a pin through the tracing paper into the rotation centre. Then trace the figure.



Step 3: Turn the tracing paper 180° clockwise around the rotation centre. Mark the vertices of the figure on the tracing paper by pressing hard with a pencil.



Step 4: Remove the tracing paper and pin. Draw the rotation image by connecting the impressions of the vertices that were marked. Label the vertices of the rotation image.

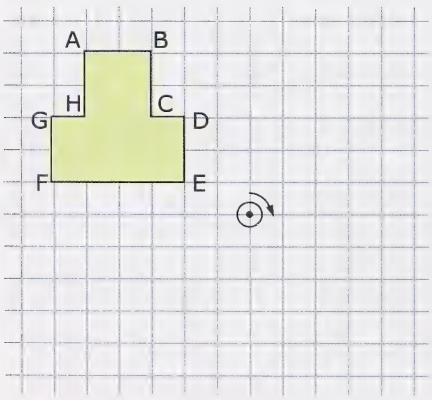


After you have drawn several rotation images, you may be able to draw the turn image without using tracing paper. Sometimes you will be able to visualize the turn. If not, just continue to use tracing paper. You can even use a clear plastic bag for tracing.

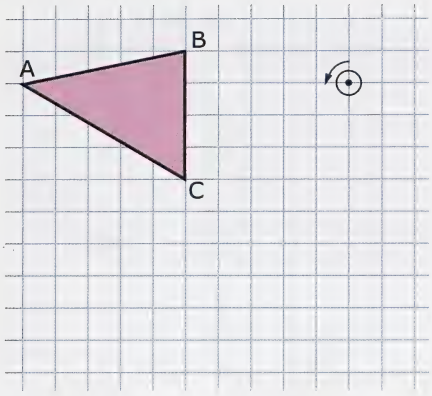
Self-Check

SC 4. Copy both of the following polygons onto graph paper. Then draw and label the rotation image of each figure for the given turn centre and arrow.

a.



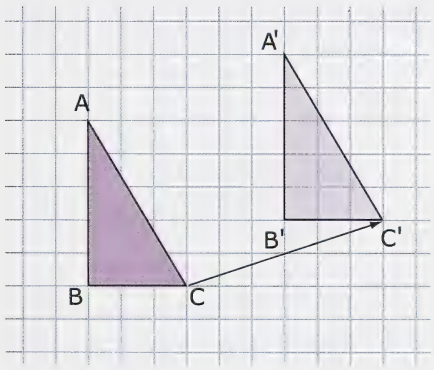
b.



Compare your answers in the Appendix.

When talking about geometric figures, a translation is a motion in which a figure is moved or made to slide in a straight line from its original position to a new position. A translation arrow indicates the direction and distance of the translation.

Look at this translation of the triangle.

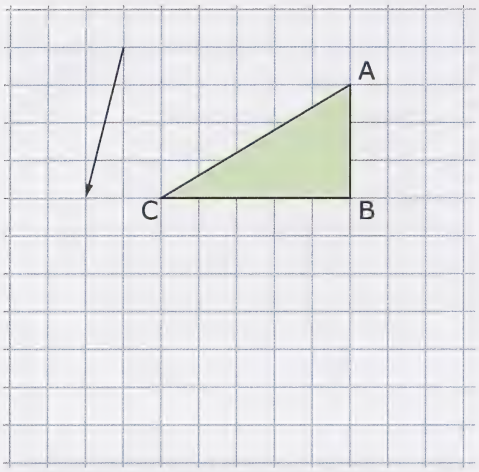


Eyewire/Getty Images

In this translation, vertex A moves to A′, B moves to B′, and C moves to C′ along a line segment parallel to the translation arrow. The vertices move six units right and two units up.

Example: Draw the Image of a Translation

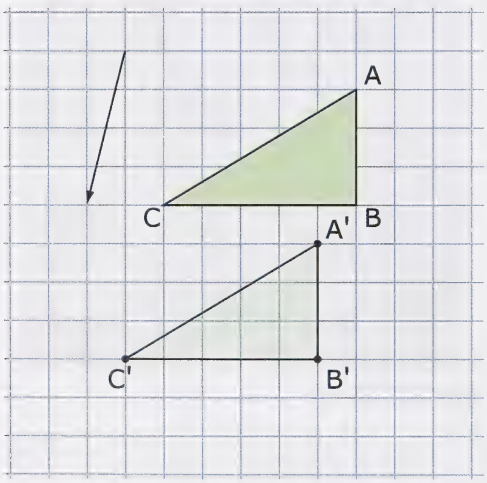
Draw the translation image of the triangle according to the slide arrow beside the triangle.



Solution

The translation arrow indicates that the triangle is to move one unit left and four units down. Mark the image of each vertex by moving one unit left and four units down from the vertex in the given image. Label the vertices for the image with the prime notation.

Connect the vertices to complete the drawing of the image.



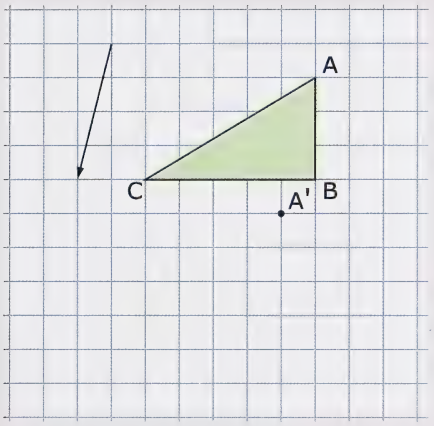
$\Delta A'B'C'$ is the translation image of ΔABC.

The solution may be easier to follow if you see it broken down into more detail. To do that, read the following My Guide.

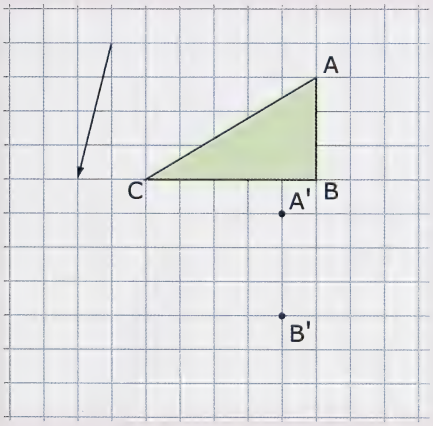
My Guide

Step 1: Describe the rule for the translation arrow. The translation rule is one unit left and four units down.

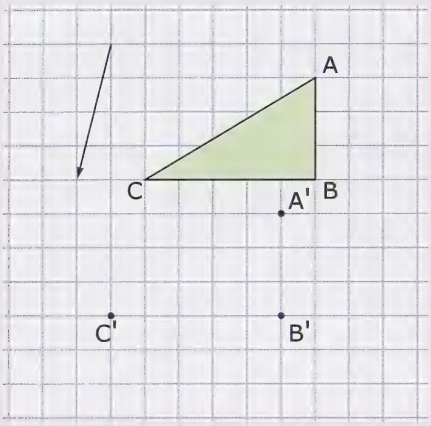
Step 2: Put your pencil tip on A. Count left one and down four. Put a dot at that position and label it A′.



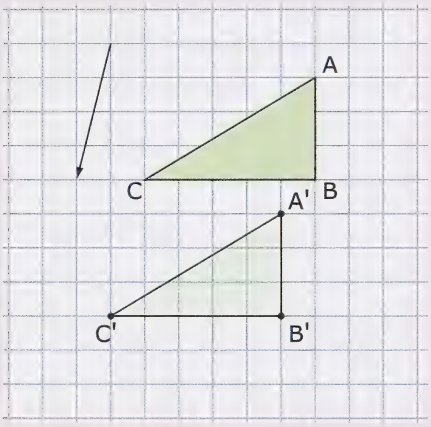
Step 3: Put your pencil on B. Count left one and down four. Put a dot on that position and label it B′.



Step 4: Put your pencil on C. Count left one and down four. Put a dot on that position and label it C′.



Step 5: Join the dots to show the translation image.

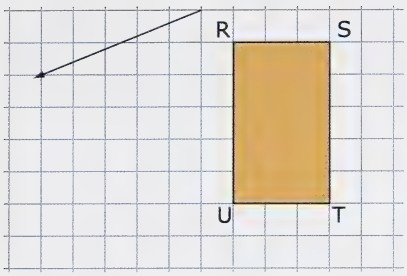


ΔA′B′C′ is the translation image of ΔABC.

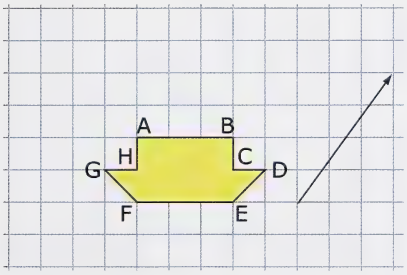
Self-Check

SC 5. Copy both of the following polygons onto graph paper along with their translation arrows. Use words to indicate the directions of the slide arrow. Then draw and label the translation image according to the given slide arrow.

a.



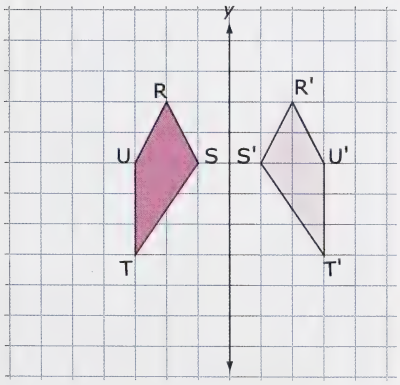
b.



Compare your answers in the Appendix.

In the study of polygons, a reflection is a transformation in which a polygon is flipped or reflected over a mirror line.

A quadrilateral and its reflected image are shown on a grid. The mirror line for the reflection is included.

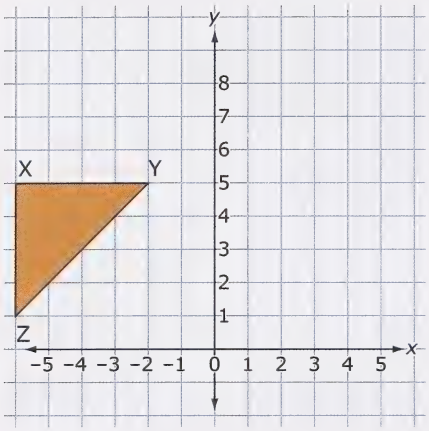


The vertices of the original quadrilateral are labelled R, S, T, and U. The vertices of the slide image are labelled R′, S′, T′, and U′.

Notice that the figure and its reflected image are on opposite sides of the mirror line. The matching vertices are the same distance from the flip line.

Example: Draw a Reflected Image

Draw the reflected image. The original vertices are X (–6, 5), Y (–2, 5), Z (–6, 1). The *y*-axis is the mirror line for this reflection.

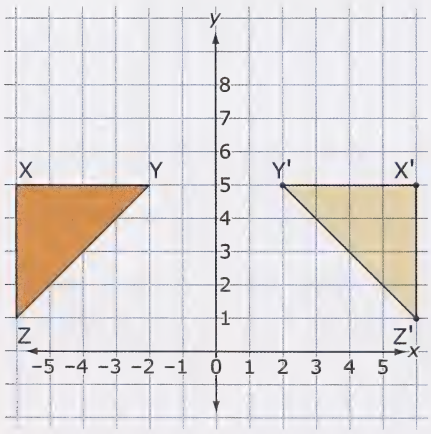


Solution

The reflected image of the vertices X (–6, 5), Y (–2, 5), Z (–6, 1) are these:

X' (6, 5), Y' (2, 5), and Z' (6, 1)

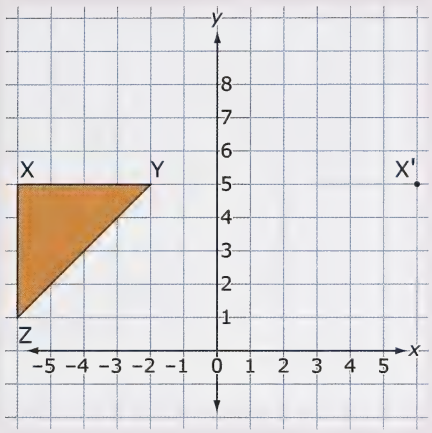
So the reflected image of the triangle is in the triangle in the first quadrant:



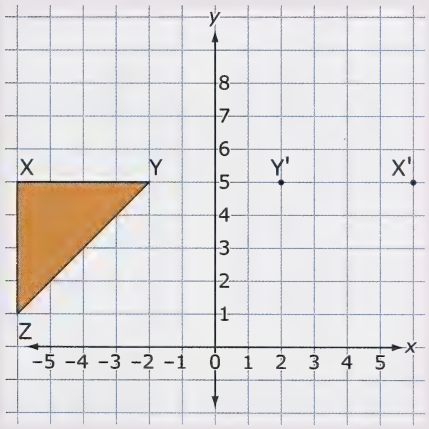
$\Delta X'Y'Z'$ is the reflected image of ΔXYZ.

My Guide

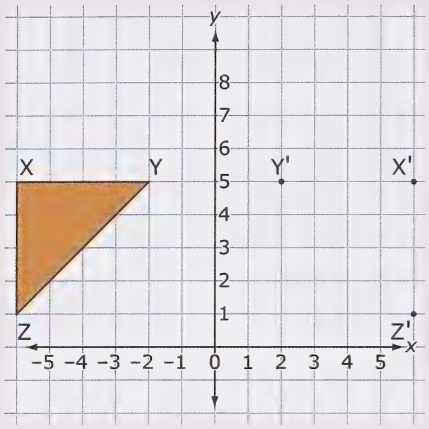
Step 1: Put your pencil on X. Count the number of units that X is from the mirror line (the *y*-axis). Count the same number of units on the other side of the mirror line and make a dot. Label it X′.



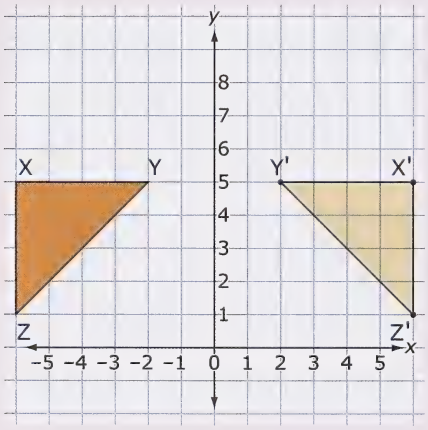
Step 2: Put your pencil on Y. Count the number of units that Y is from the mirror line. Count the same number of units on the other side of the mirror line and make a dot. Label it Y′.



Step 3: Put your pencil on Z. Count the number of units that Z is from the mirror line. Count the same number of units on the other side of the mirror line and make a dot. Label it Z′.



Step 4: Connect the dots to show the reflected image.

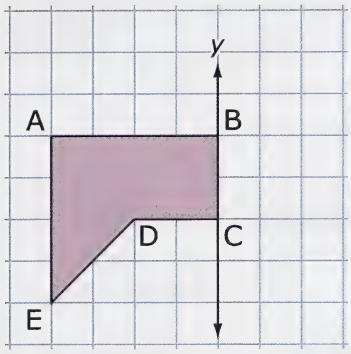


MathLinks 8: Solutions CD (Toronto: McGraw-Hill Ryerson, 2008) 167. Reproduced by permission.

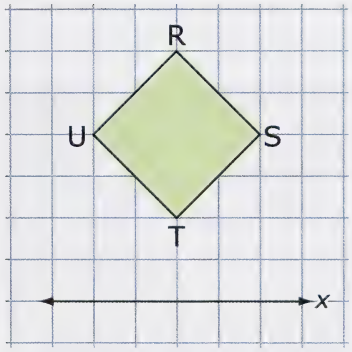
Self-Check

SC 6. Copy both polygons onto graph paper. Then draw and label the reflected image according to the given mirror line.

a.



b.



My Guide

You may want to place an x-axis and a y-axis on each grid. You'd put the y-axis on the vertical mirror line in part a. and the x-axis on the horizontal mirror line in b. Then you can use coordinates to find the images of vertices.

Or you can simply count the number of squares each vertex of the original polygon is from the mirror line and place the image of this vertex the opposite side of the mirror line and an equal number of squares away from the mirror line.

Compare your answers in the Appendix.

Read

Do you remember the important information from this lesson? Read "Key Ideas" on page 168 of your textbook for a short summary of the lesson's main ideas. One additional main idea is this: Transformations include rotations, translations, and reflections.

Make notes about this summary for your future study purposes. Creating a foldable like the one shown on page 162 of your textbook is an enjoyable method for creating your notes.

Try This

TT 6. Complete "Communicate the Ideas" questions 1 and 2 on page 168 of your textbook.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 7. Complete "Check Your Understanding" questions 3.a), 3.c), 4, 5, and 7 on pages 168 and 169 of your textbook.

Compare your answers in the Appendix.

SC 8. Solve "Apply" questions 8 and 9.b) on page 169 of your textbook.

My Guide

For question 8, you are to choose two 3-D objects. Avoid objects that have many curves or surface detail. The front, side, and top views may be too difficult to draw. You need not restrict yourself to objects that you find in a classroom. An object you have at home is also fine.

Compare your answers in the Appendix.

Extra Practice

Feeling challenged by the ideas presented in the lesson? More practice might help you become comfortable with the ideas. Turn to pages 168 and 169 in your textbook, complete additional questions from "Check Your Understanding" and then check your work using the answers given on page 490 at the back of your textbook. If you still have questions, be sure to contact your instructor for additional help.

Assignment

Go to the Unit 4 Assignment Booklet, and complete "Unit 4: Lesson 1 Question Set."

Hold on to your work. You will submit your work from this lesson according to instructions in one of the lessons that follow. So save your answers to this question set to your Math 8 folder.

Going Beyond

Think you're smarter than a bumblebee? Show the path that the bee should follow along the edges of a cube by completing "Extend" question 11 on page 169 of your textbook. You'll be able to apply your visualization skills with 3-D objects in the context of solving a pathway problem.

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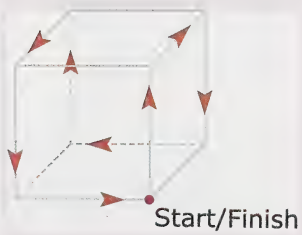
My Guide

The bee does not have to travel along each edge. But after the bumblebee starts from one vertex, it must visit every other vertex once and only once. Then it must return to its starting vertex.

You may want to trace the bumblebee's path along an actual cube. Otherwise, draw a diagram of the cube with the back edges shifted so that no edge "covers up" another. Then the line segments of the diagram represent possible paths the bee should take.



Compare your answers in the Appendix.

Lesson Summary

In this lesson you learned to draw the top, front, and side views of 3-D objects. Some of these objects you built, others you saw on isometric dot paper, and some actually existed in your classroom or home. You also learned to build and draw 3-D objects on the basis of their top, front, and side views. You also reviewed transformations of 2-D figures—the rotation, translation, and reflection of polygons. Although 2-D figures and 3-D objects may look different after rotation, they do not change in shape or size.

From your experiences in this lesson, you found an answer to this question: How do you draw a three-dimensional object on a flat surface? You learned that drawing three different views of the object is one way to answer the question. The other method is to make a drawing of the object on isometric dot paper.

You started out this lesson by thinking about the stacked goods on shelves of a store. The view that customers have of the packages is affected by how the packages are positioned on the shelf. Having the best face of the package as the one viewed by store customers is a good practice. Having the best face forward makes the product more appealing; so the product is more likely to be sold.

Have you started your Unit Problem? Refer to the Unit 4 Summary and begin part 1 of the Unit Problem.

In the next lesson you will see how to make a 3-D object from a flat piece of material that has some fold marks and is of the right size and shape.

Lesson 2: Nets of Three-dimensional Objects

Get Focused

Ships carry cargo in shipping containers rather than directly in the hull. These shipping containers are the size of a semitrailer. If you own a product manufactured outside of North America, then it likely travelled most of the journey to your house in a shipping container.

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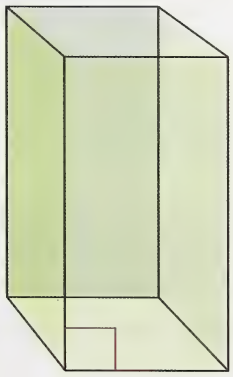
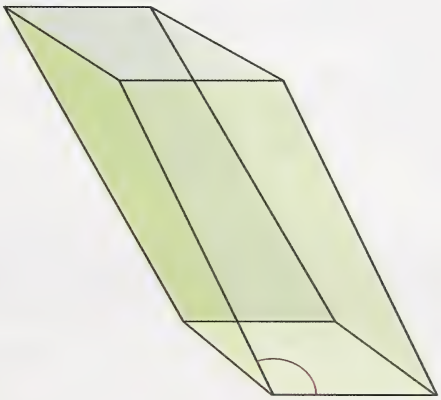
There is a great picture of shipping containers on page 170 of your textbook *MathLinks 8*. You can see that a shipping container is shaped like an elongated cardboard box but is much bigger. Standard shipping containers are over 6 m long. They measure 6.1 m by 2.4 m by 2.6 m.

Study the margin features on page 170. You can see a diagram of a **rectangular prism**. In this lesson you will read and hear about right prisms, and it is important to know what makes a rectangular prism a *right* rectangular prism.

rectangular prism: a prism that has six rectangular faces



90° angle
between base and height



Not a 90° angle
between base and height

© 2009 Alberta Education

right prism: a prism having sides that are perpendicular to its base

net: a two-dimensional shape that, when folded, encloses a three-dimensional object

The diagram shows two rectangular prisms. One is a **right prism** but the *leaning* one is not. In this study of prisms, you will focus on right prisms. Familiar objects, such as a package of butter, an alphabet block, a typical cardboard box, and a tissue box, are right rectangular prisms. So is the standard shipping container. To have a right rectangular prism, the base and any lateral face must be at a right angle (90°) to each other.

A flat shape that depicts a 3-D object, often with lines drawn on it, is called a **net** of the 3-D object. A 3-D object as large as a shipping container can be represented by a net.

In this lesson you will draw nets for 3-D objects and construct 3-D objects from their nets.

This lesson will help you answer the following critical question: How does a net represent a three-dimensional object?

In this lesson you will need grid paper. You may use "1-cm grid paper" on the Math 8 Multimedia DVD. You may find it helpful to go to the Math 8 Multimedia DVD and use "Square Dot Paper" to complete the Unit 4: Lesson 2 Question Set. As well, you will need a pair of scissors, some clear sticky tape, and some examples of small rectangular prisms such as blocks of wood and cardboard boxes.

As in the previous lesson, you may use wooden or plastic building blocks, alphabet blocks, or even sugar cubes to make various 3-D objects. **Caution:** Keep small objects out of the reach of babies or small children.

Assignments

Your assignment will consist of the following:

- posting and responding to the discussion board
- adding to your Math 8 folder
- completing Unit 4: Lesson 2 Question Set

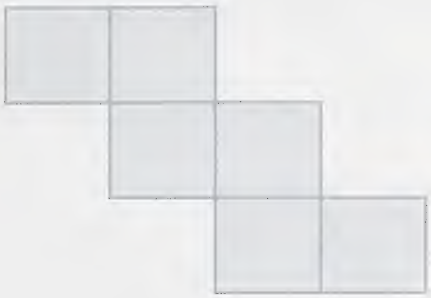
Explore

Can you imagine what the net of a cube would look like? If you took a paper cube and unfolded it, what would the flattened cube look like? Check out the diagram in the margin of page 170 of the textbook to see a possible net for a cube.

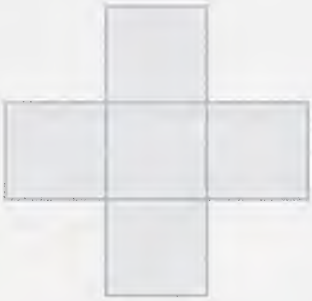
Self-Check

SC 1. A cube can be made by various nets that look quite different. A student drew these as nets for a cube.

Net A



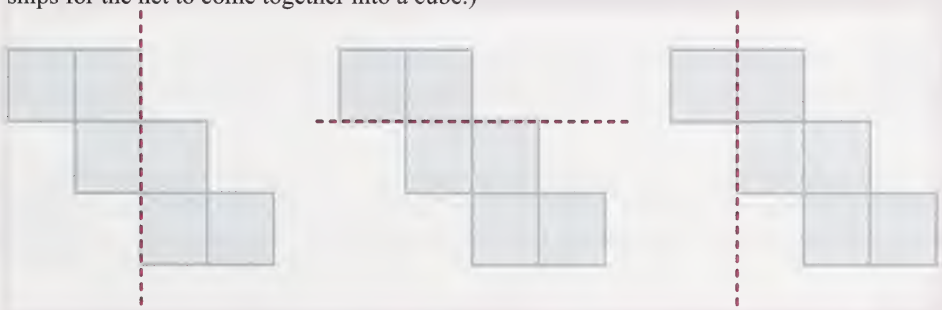
Net B



Which of the nets would actually work as a net for a cube? Give reasons for your answer.

My Guide

A net requires that it has the same number of faces as the 3-D object it is supposed to represent. Net A has six faces, which is the same number of faces necessary to build a cube. You may want to cut out the net and try folding it to show that is does work as a net for a cube. (Remember, all of the lines do not have to be attached. You might need to make some snips for the net to come together into a cube.)



Now focus on Net B. Decide whether it has the same number of faces as the cube it is supposed to represent.

Compare your answer in the Appendix.

In the "Explore the Math" activity presented on pages 170 and 171 of your textbook, you'll try to build a right rectangular prism from some flat shapes. This experience will help you to identify the correct nets for a given 3-D object.

Work with a partner as you do the Self-Check questions. Your partner may be assigned by your teacher or be a friend or family member. Working with a partner will allow you to discuss predictions and share the cutting and taping involved to construct right rectangular prisms.

You and your partner will each need grid paper on which to copy shapes. The 1-cm grid paper will work well. Also have on hand a pair of scissors and some clear sticky tape. Blocks of wood, small cardboard boxes, or unit blocks will be helpful when you are asked to make your own nets for a right rectangular prism.

SC 2. Complete "Explore the Math" questions 1, 2, 3, 4, and 5 on pages 170 and 171.

Compare your answers in the Appendix.

Discuss and Share

Get out your digital camera and share, with another student on your discussion board, the nets you drew for the right rectangular prism of "Explore Your Math" question 5. Discuss how your nets compare and the number of ways a net can be drawn for the 3-D object.

You may communicate by some form of online chat. React to each other's ideas; do not just present information without listening to the other participants. Promote conversation by using expressions such as "What do you think about . . . ?" Use your discussion to decide whether or not there is more than one way to draw a net for a 3-D object.

Try This

TT 1. There are many different possibilities for drawing nets. Note how your classmates or partners drew nets that are different from yours by completing "Explore the Math" question 6 on page 171. Use your findings from the previous Discuss and Share to answer question 6.

Place a copy of the final version of your answers in your Math 8 course folder.

Connect

Let's now build on what you and your partner discovered during the completion of the Explore section.

Read

When do you think you would use nets outside of the classroom? You can probably think of many examples. Have a look at "Example 1: Draw a Net for a Three-Dimensional Object" on page 171 of your textbook to see how you can draw a net for an umbrella stand. Look at the net to identify the part that will remain flat.

Self-Check

© Jerome Scholler/shutterstock

SC 3. Complete "Show You Know" at the bottom of page 171 to draw a net. As you do this question, envision the soup can as having no lip at the top and bottom and no ripples in its curved side.

Compare your answers in the Appendix.

The net you made for the unopened soup can be used to make a right **cylinder**.

> **cylinder:** a prism having two parallel bases that are circular and the same size
>
> A cylinder has two circular bases and one curved surface connecting the circular bases.

Part of the net is a rectangle. Maybe you need to prove for yourself that the curved surface of a soup can forms a rectangle when the can is flattened. If you want to see that for yourself, peel the label off a soup tin. Unroll the label. Then you should see a rectangle.

Watch and Listen

By watching "Cubes" on the Math 8 Multimedia DVD, you will be able to see how a cylinder can be unfolded into its net. In "Cubes," pick the cylinder as the shape. Then press "Unfold" to unfold (and unroll) the cylinder into its net. You may want to pick another shape for comparison.

Read

Pup tents have the shape of a **triangular prism**. Read "Example 2: Build a Three-Dimensional Object From a Given Net" on page 172 of your textbook, and think of the cutout shape as the fabric part of a tent. Follow the example to find out whether the fabric can be folded to make the shape of a pup tent.

triangular prism: a prism having two parallel faces shaped like triangles of the same size and shape

My Guide

Think of a pup tent—without a door—as you work through the textbook example.

© 2009 Jupiterimages Corporation

In following the example, you may want to print out BLM 5-8 Example 2 instead of tracing the net shown in the example. "BLM 5-8 Example 2" is on the Math 8 Multimedia DVD.

Self-Check

SC 4. Complete "Show You Know" at the bottom of page 172 of the textbook. Before building the object, try to predict what the object will look like. You may go to the Math 8 Multimedia DVD and print out the net for "Example 2: Show You Know."

Compare your answers in the Appendix.

Read

Do you remember the important information you have learned in this lesson so far? Read "Key Ideas" on page 173 of your textbook for a short summary of the lesson's main ideas—how to draw nets and how to use nets to build 3-D objects.

Make notes about this summary for your future study purposes. You might want to add these notes to a foldable like the one shown on page 162 of the textbook.

Try This

TT 2. Now that you have had a little experience with nets, try a few questions in the textbook to see if you can accurately use your new knowledge. Apply the knowledge and skills relating to nets of 3-D objects as you answer the following questions. Complete "Communicate the Ideas" questions 1 and 2 on page 173 of your textbook. For question 2, you may *describe* how you would show your help.

My Guide

Don't simply assume that a net is correct if it has the correct number, size, and shape of faces. The faces have to be connected appropriately for the 3-D figure you want to make.

Place a copy of your answers in your Math 8 course folder.

Watch and Listen

Go to the Math 8 Multimedia DVD, and open the piece titled "Nets and 3-D Objects." The multimedia piece "Nets and 3-D Objects" provides you with an activity where you observe a 3-D object change into an appropriate net.

Follow these steps:

- When you select the title, you will be presented with two tabs. Choose the tab titled "Use It 1: Nets."
- Then select one of the following: a rectangular prism, a triangular prism, or a cylinder from the six objects.
- When you move the slider to the right, you will see the surface of the 3-D object unfold into an appropriate net of the object.
- Once you see how the 3-D object unfolds into its net, select the most appropriate net from the three choices provided at the bottom of the window.

You will receive a check mark for a correct response. Continue with the selection of the remaining two objects: a rectangular prism, a triangular prism, or a cylinder. **Note:** You do not need to select the cone or the pyramids. These 3-D objects are not part of this lesson. Select them only if you want to explore these 3-D objects.

Self-Check

SC 5. Test your skills with nets by completing "Check Your Understanding" questions 3, 4, 6, and 7 on pages 173 and 174 of your textbook.

SC 6. Complete "Check Your Understanding" question 8 on page 174 of your textbook.

Compare your answers in the Appendix.

Extra Practice

Turn to pages 173, 174, and 175 of the textbook. For extra practice, you may complete additional questions from "Check Your Understanding." Then compare your work with the answers given on pages 490 and 491 of the textbook.

Assignment

Go to the Unit 4 Assignment Booklet, and complete "Unit 4: Lesson 2 Question Set."

You may find it helpful to use "Square Dot Paper" to complete question 3. You can find "Square Dot Paper" on the Math 8 Multimedia DVD. Submit your answers to the Lessons 1 and 2 Questions Sets to your teacher for evaluation.

Going Beyond

Most board word games (e.g., Scrabble®) are played in two dimensions—that is, on a flat surface. It is possible, however, to make a game in which you can place tiles on top of tiles already on the board. The stacks form 3-D objects.

Turn to page 175 of your textbook, and answer "Extend" question 11. You'll be able show your visualization skills with 3-D objects that are made by playing a word game.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you discovered how nets show the shape of 3-D objects in a 2-D form. You also learned how to draw a variety of nets for the same 3-D object. As well, you constructed 3-D objects from one or more of the nets that produced it. Additionally, you found that a net can accurately represent the 3-D object when it is folded appropriately—that is, with no holes or overlap.

In the next lesson you will relate the area of a net to the surface area of the 3-D object is represents.

Remember to continue your work on your Unit Problem. Refer to Unit 4 Summary, and work on part 1.

Unit 4: Polygons, Prisms, and Cylinders

Lesson 3: Surface Area of a Prism

Get Focused

Many food containers are in the shape of right rectangular prisms. This shape allows containers to stack easily. The packaging keeps food contents together and protects the contents from contamination. The packaging also provides a surface for labels and other information about the contents. The amount of information that can be placed on the packaging depends on the surface area of the container.

This lesson will focus on determining the surface area of right rectangular prisms and right triangular prism. There will also be problems to solve involving the surface area of such prisms.

This lesson will help you answer the following critical questions:

- How can nets be used to find the surface area of a prism?
- How does the area of a net represent the amount of material used for packaging?

In this lesson you will need a ruler, a pair of scissors, and empty cardboard box. The cardboard box should be made of thin cardboard. For example, an empty cereal box could be used.

Assignments

Your assignment will consist of the following:

- posting and responding on the discussion board
- adding to your Math 8 folder
- completing Unit 4: Lesson 3 Question Set

Explore

In the "Explore the Math" activity presented on page 176 of your textbook, you will take apart the packaging of a prism-shaped container. Unfolding the packaging on a table top allows you to determine the area of the package. As well, you also see how the area of the unfolded packaging relates to the area of the net and the **surface area** of the prism-shaped container.

surface area: the number of square units needed to cover a 3-D object; also, the sum of the areas of all the faces of a 3-D object

Since surface area is based on the sum of areas, you need a firm understanding of area. The multimedia piece titled Area may be a good review. This multimedia item allows you to make rectangles of different sizes and shapes on a 1-cm grid. Just drag a corner—the red dot—of the rectangle to different positions. Watch how perimeter and area calculations change as you make different rectangles. If you want the challenge, try to hit the target perimeter and area values. When you hit the target values and unclick, the "New" button lights up. Clicking the "New" button will present new targets.

Have a ruler, a pair of scissors, and an empty cardboard box on hand.

Work with a partner as you do this Explore activity. Your partner may be assigned by your teacher or be a friend or family member. Discuss and share what your nets look like and ways to determine the surface area. You may also share digital pictures to show your cardboard box and the net you made from it.

Discuss a how a change in each of the dimensions of a 3-D object affects the shape of the net.

Try This

TT 1. Follow the directions in question 1 of "Explore the Math" on page 176. Your unfolded packaging as a proper net should not have holes in it or unnecessary cuts.

My Guide

Q: Why are flaps often included in the packaging?

A: The flaps allow the sides of the material to be attached to form edges.

Q: Should the area of hidden flaps be added to the total area of the net when considering the area of the outside of a box?

A: Hidden flaps do not form part of the outside of the box. You can see that if you just hold the net together to form the original box.

Q: Suppose flaps are not hidden but exposed on the outside of the box. Should the area of these flaps be counted in calculating the area of the outside of the box?

A: If the flaps are attached to the outside, then you can consider them part of a face. Then the areas of the flaps will already be accounted for in the areas of the faces. The important idea is that you don't count areas of overlap twice. So you should not count areas of flaps—whether hidden or exposed—in addition to the areas of the faces.

TT 2. Complete question 2 of "Explore the Math" on page 176 to develop a strategy for determining the surface area of the cardboard box you cut open to create its net. Compare your strategy with your partner's strategy. Try to come to some agreement with your partner about how to find the surface area of the cardboard box.

My Guide

Q: It's easy to forget a face in the calculations of surface area of a rectangular prism. How can you tell that you've got all of the faces are included in the calculation?

A: A rectangular prism has six faces. Your calculations should account for the areas of all six of them.

TT 3. Now reflect on your thinking by completing "Explore the Math" question 3 on page 176.

Discuss and Share

Discuss with someone you know from your discussion board how your packaging nets compare and the number of ways to find the surface area of a prism-shaped package. Are there any characteristics of prisms that help you in determining its surface area?

You may communicate by some form of online chat. React to each other's ideas, rather than just presenting information without *listening* to the other participant. Promote conversation by using expressions such as "What do you think about . . . ?" or "How did you . . . ?"

Use the discussion to revise your answer, as needed, to attain your best work.

Place a copy of the final version of your answers in your Math 8 course folder.

Connect

Now build on what you and your partner discovered while you completed the questions in the Explore section.

Read

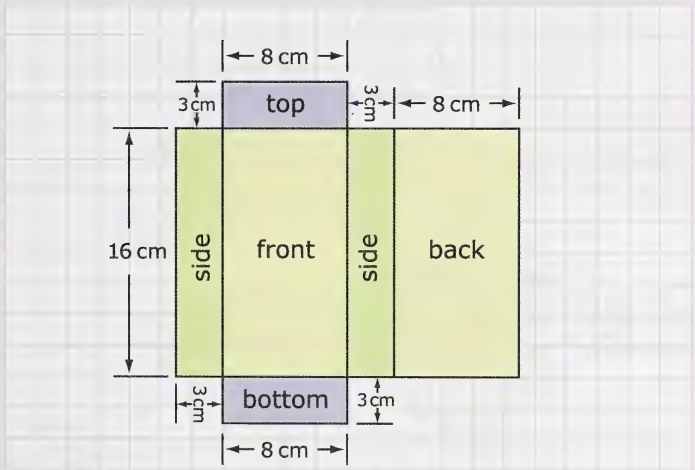
"Example 1: Calculate the Surface Area of a Right Rectangular Prism" on page 177 of your textbook shows how to find the surface area of a rectangular prism. Did you find the area of your cardboard box using a similar approach? Pay close attention to how the net of the prism is used to keep track of the six faces and their dimensions. Instead of adding the areas of six faces, the textbook example shows how it can be done another way—by finding the area of the three sets of opposite faces. The values of these three faces are doubled by multiplying by 2 and then adding the three values together to determine the total surface area of the prism.

Self-Check

SC 1. Complete "Show You Know" at the top of page 178. You may want to create the net to help you visualize the rectangular prism, or you might want to draw the three pairs of sides in creating this prism shape.

My Guide

Sketching a net for the object can help you visualize the faces and their areas.



Compare your answers in the Appendix.

Read

When finding the surface area of a rectangular prism, you must account for all the faces—all six of them. A triangular prism has less faces.

Do you know how many faces are on a triangular prism? Check out "Example 2: Calculate the Surface Area of a Right Triangular Prism" on pages 178 and 179 of your textbook. See how to find the surface area of a triangular prism. Notice how many faces a triangular prism has and see how that number makes the calculation of surface area a little bit different.

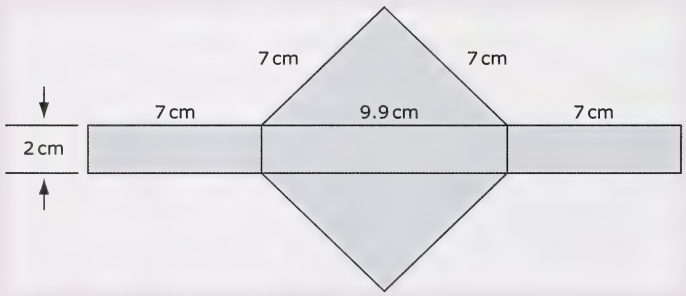
Self-Check

SC 2. Complete "Show You Know" near the middle of page 179 of your textbook. Notice how the triangle is different than the one shown in the example, because the base of the prism is a right triangle—the 7-cm side is also the height of the triangular base.

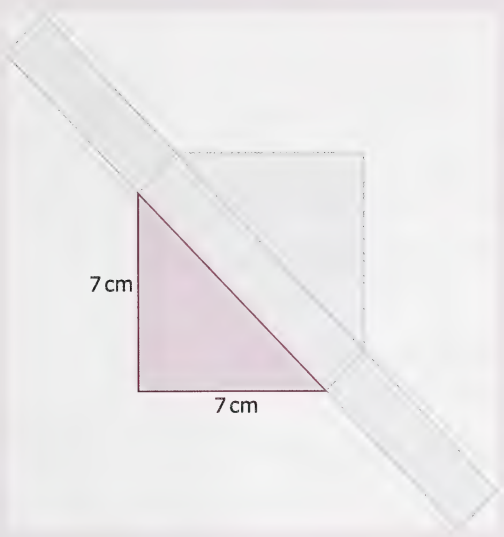
My Guide

The triangles at the end of the prism are isosceles triangles, not equilateral triangles. So the three rectangular faces of the prism are not all the same size.

A net for the triangular prism can show you the different faces and can help you keep the faces and their dimensions straight.



The triangular faces are right triangles. When you view the net from an angle, you can see that the base and the height measure 7 cm.



Compare your answers in the Appendix.

Read

Do you remember the important information you have learned from this lesson so far? Read "Key Ideas" on page 179 of your textbook for a summary of the lesson's main idea—the surface area of a prism is the total area of all its faces.

Now is a great time to make a note about this summary for your future study purposes. You might want to add your notes to a foldable like the one shown on page 162 of the textbook. Add diagrams to go with your information to help you remember the key ideas that were presented.

Try This

TT 4. Apply the knowledge and skills related to surface area. Complete "Communicate the Ideas" question 2 on page 179 of your textbook.

Place a copy of your answers in your Math 8 course folder.

Self-Check

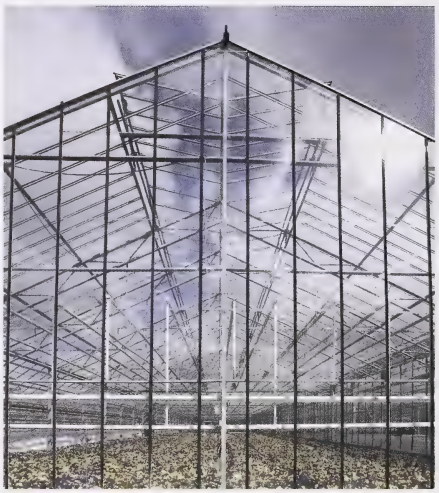
SC 3. Complete "Check Your Understanding" questions 3 and 5 on page 180 of your textbook.

SC 4. Complete "Check Your Understanding" questions 7, 8, 10, and 12 on pages 180 and 181 of your textbook.

Compare your answers in the Appendix.

Extra Practice

Do you feel as though you have a good understanding of how to calculate the area of a prism? Were you able to answer five of the questions above correctly? If so, continue on to the assignment. If not, turn to pages 180 and 181 in your textbook. For extra practice, complete additional "Check Your Understanding" questions. After each question, compare your work with the answers on page 491 of the textbook. When you feel ready, complete the assignment.



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A greenhouse is a structure made of glass walls used to grow plants. The glass walls allow in the Sun's rays while protecting the plants inside.

Assignment

Go to the Unit 4 Assignment Booklet, and complete "Unit 4: Lesson 3 Question Set."

Save your answers to this question set. Your answers are to be submitted according to the instructions in one of the lessons that follow.

Going Beyond

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In the next question you will be challenged to find the cost of painting a rectangular room. The walls of the room are like the faces of a prism, so calculating the painting area is like finding the area of a face of a prism.

Remember to consider which walls are going to be painted and which are not as you carry out your calculations. What calculation will you use to determine how many litres of paint to purchase?

Turn to page 181 of your textbook, and answer "Check Your Understanding" question 16.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you worked out strategies to determine the surface area of right-rectangular prisms and right-triangular prisms. You learned that there were tips you could use to help speed up the calculation process. You also gained experience solving problems involving surface area.

Remember to continue your work on part 1 of your Unit Problem. Refer to the Getting Started section of the Unit 4 Summary.

In the next lesson you will explore the surface area of 3-D objects known as cylinders.

Unit 4: Polygons, Prisms, and Cylinders

Lesson 4: Surface Area of a Cylinder

Get Focused

Glow sticks are fascinating products. They come in various colours and give off light without using electricity. Instead, the energy needed to light the glow stick comes from a special chemical reaction that gives off light but, curiously, hardly any heat. A glow stick can be used as a baton to control traffic, a flashlight for scuba diving, or just for fun as bracelets that glow.

© Sébastien Fouque/Fotolia LLC

A glow stick is shaped like a length of garden hose. It can be bent to form a bracelet. When straight, a glow stick has the cylindrical shape—the same shape as many other products or packages such as a tin can, a roll of paper towel, a candle, and a roll of candy.

The outside of a glow-stick bracelet is made of plastic cut out of large flat sheets. How much plastic is needed depends on the area of the sheet of plastic that must be used up to form the outside surface of the glow stick. The area of plastic needed is equal to the surface area of the cylinder-shape of the glow stick.

In this lesson you will determine the surface area of cylinders. You will also solve problems involving the surface area of cylinders.

This lesson will help you answer the following critical question: How can a net and a formula be used to find the surface area of a cylinder or a cylindrical object?

In this lesson you will need 1-cm grid paper and a handheld calculator. You can find "1-cm Grid Paper" on the Math 8 Multimedia DVD..

Assignments

Your assignment will consist of the following:

- posting to your discussion board
- adding to your Math 8 folder
- completing Unit 4: Lesson 4 Question Set

Explore

In the "Explore the Math" activity presented on pages 182 and 183 of your textbook, you will look at how to determine the area of plastic sheet needed to make a glow stick 21 cm long. In essence, this problem is like finding the surface area of a glow stick long enough to fit around a person's wrist.

Watch and Listen

In order to find the surface area of a glow sick, you will need to recognize the components that comprise its cylindrical shape. The surface area of a cylinder can be pictured from its net.

Go to the Math 8 Multimedia DVD, and watch "Cubes" to see how a cylinder can be unfolded into its net. Under the heading "Pick a shape," choose the cylinder. Then press "Unfold" to unfold (and unroll) the cylinder into its net. You may want to pick another shape for comparison. Only one other shape needs unrolling. The other two shapes just need unfolding.

Discuss and Share

Work with others on the discussion board or in your class as you complete the following Self-Check question. If your partner is not with you, communicate with instant messaging as you work through the questions. A calculator will be handy.

Self-Check

SC 1. Complete "Explore the Math" questions 1, 2, and 3 on pages 182 and 183 in order to find how to determine the surface area of the glow stick. For question 1, draw your net on 1-cm grid paper. (You can find "1-cm Grid Paper" on the Math 8 Multimedia DVD.) For question 3, also indicate the amount of plastic needed to make the glow stick.

My Guide

Q: How can you use the strategies you developed for calculating the surface area of rectangular and triangular prisms to help find the surface area of a cylinder?

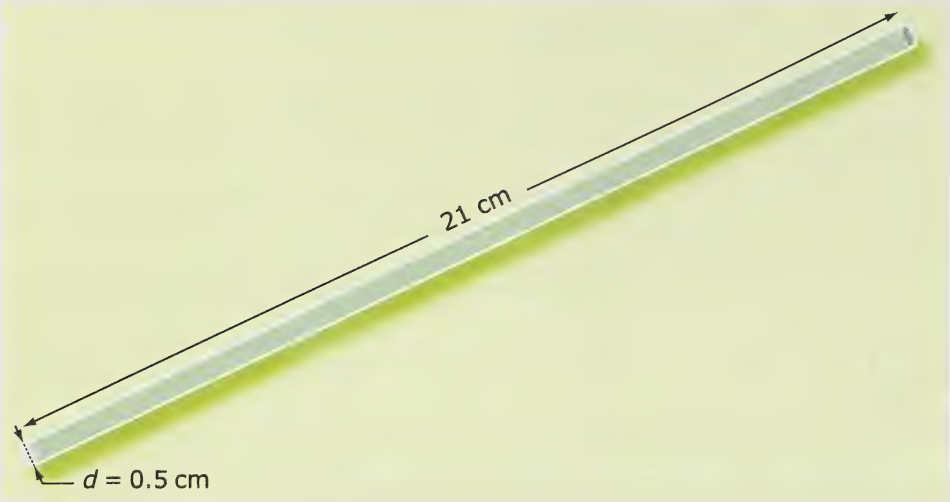
A: In the strategies used earlier, you had to account for the areas of all of the faces. You will have to account for all the faces of a cylinder too. Remember that a cylinder has one face that is a curved surface. You can keep track of the faces by making a net.

Q: What shapes should make up your net?

A: There should be two circles for each end of the glow stick, and a rectangle for the curved face of the glow stick.

Q: How long should the rectangle shape be made?

A: The length should be equal to the height of the cylinder.

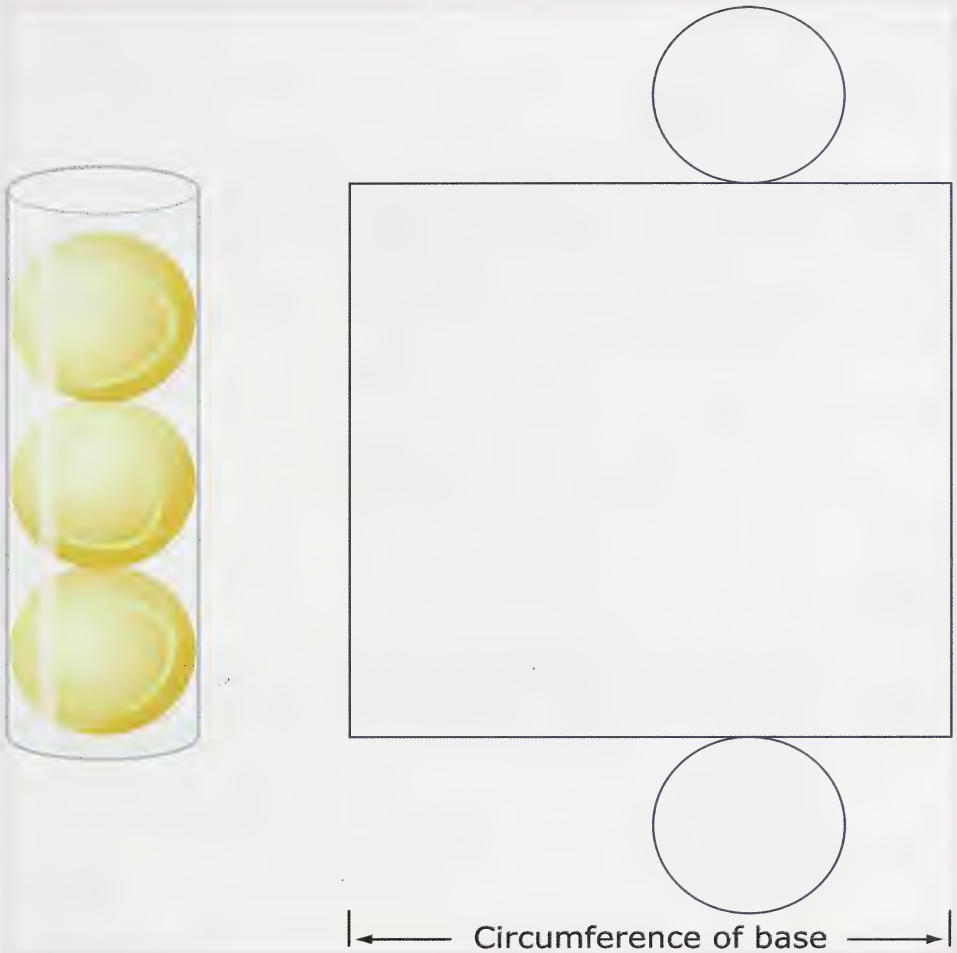


Q: How wide should the rectangle be?

A: The width of the rectangle should be equal to the circumference of a circle at the end of the glow stick.

If you don't see that relation between width and circumference, think of the label surrounding a tennis ball container, for example. When you unroll the container to remove the label, the container makes one rotation. (You may also look again at the unrolling of the curved side of a cylinder in the multimedia piece titled Cubes.)

So, the width of the rectangle should be equal to the circumference of the circle at either end of the container. Of course if the label overlaps somewhat, the label will be a bit wider than the circumference.



Q: How do you find the area of the circular part of the net?

A: You apply the formula $A = \pi r^2$.

Compare your answers in the Appendix.

Try This

TT 1. Now take a crack at "Explore the Math" questions 4 and 5 on page 183. For question 5, check whether you got 33.37 cm^2 as the surface area of the cylindrical glow stick. If your answer is off this value, what do you think caused the difference? Did you make a simple error, or was your strategy not effective?

Discuss and Share

Discuss your strategy with someone you know from your discussion board. Compare the strategies you both used. Promote conversation by using expressions such as "What do you think about my strategy involving nets?" and "Would this strategy work for any right cylinder?"

Use your discussion to improve your answer. Place a copy of the final version of your answers to TT 1 in your Math 8 course folder.

Connect

Now build on what you and your partner discovered during the Explore.

Read

"Example 1: Determine the Surface Area of a Right Cylinder" on pages 183, 184, and 185 of your textbook shows how to find the surface area of a cylinder. See how your knowledge about the circumference and area of a circle is used in finding the surface area. Make a note of the formulas for circumference and area of a circle.

Watch how the net of the cylinder is used to keep track of the three faces and their dimensions. Also pay attention to the additional explanations presented in balloons.

Self-Check

SC 2. Try out your skill at calculating the surface area of a cylinder by completing the "Show You Know" question near the top of page 185 of your textbook.

Compare your answers in the Appendix.

Totem poles, the monumental sculptures created by the Indigenous peoples of North America's pacific northwest coast, are famous for their beauty, grace, and artistic power. Most totem poles are created from trees that have been stripped of their bark and branches. This creates a cylindrical shape on which to carve. In planning for a totem pole, knowing how much surface is available to carve helps the artist plan what the totem pole will represent. Knowing the surface area gives the artist an idea of how much space is available for the images that the artist wants to include on the totem pole.

Read

"Example 2: Use the Surface Area of a Cylinder" on page 185 of your textbook shows how to find the surface area of a totem pole.

Self-Check

SC 3. Complete the "Show You Know" at the bottom of page 185 of your textbook.

Compare your answers in the Appendix.

Read

Do you remember the important information you learned from this lesson so far? This is a great time to stop and review. Read "Key Ideas" on page 186 of your textbook for a short summary of the lesson's main idea—the surface area of a cylinder is the total area of all its three faces.

Make notes about this summary for your future study purposes. You may well want to add your notes to your foldable like the one shown on page 162 of your textbook.

Try This

Apply the knowledge and skills relating to calculating the surface area of cylinders by answering the following questions.

TT 2. To help make sure you have a good understanding of how things fit together, complete "Communicate the Ideas" questions 1 and 2 on page 186 of your textbook.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 4. Work out the solutions to "Check Your Understanding" questions 4 and 6 on page 186 of your textbook.

SC 5. Now solve "Apply" questions 8 and 9 on page 187 of your textbook.

Compare your answers in the Appendix.

Were you able to answer the Self-Check questions without any difficulties? If so, continue on to the assignment. If you feel you need a bit more practice, turn to pages 186 and 187 in your textbook and complete additional questions from "Check Your Understanding." After finishing each question you try, compare your work with the answers given on page 491 of your textbook. Anytime you feel ready, move on to work on the assignment.

Assignment

Go to the Unit 4 Assignment Booklet, and complete "Unit 4: Lesson 4 Question Set."

Submit your answers to the Lessons 3 and 4 Question Sets to your teacher for evaluation.

Going Beyond

In this lesson you have been asked to calculate the surface area of a variety of cylinders given the necessary measurement to carry out the calculations. Now it is time to challenge your thinking about cylinders. See if you can calculate the surface area of a cylinder. The diameter and height of this cylindrical shape have to be figured out based on the objects placed inside of it.

Turn to page 187 of your textbook, and answer "Extend" question 11. Assume that the material making up the can comes in flat sheets; then the surface area of the can represents the amount of material needed to make the can. Consider any lid to be part of the can and also made from the flat sheets of material.

Compare your answers in the Appendix.

Lesson Summary

Congratulations on reaching the end of another lesson! Remember this lesson as one in which you found and described strategies to determine the surface area of right cylinders. In this lesson you also gained experience solving problems involving the surface area of right cylinders.

By drawing nets you can keep track of the areas of the faces you have to add together for the surface area of the cylinder.

This formula can be used to find the surface area of a cylinder:

$SA = 2 \times (\pi \times r^2) + (\pi \times d \times h)$

Remember to continue your work on your Unit Problem in the Unit 4 Summary. You should complete part 1 of the Unit Problem.

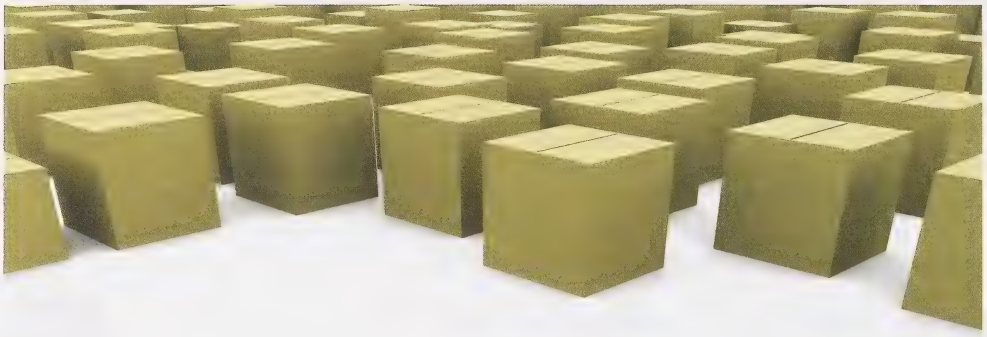
In the next lesson you will explore the amount of space a 3-D object takes up.

You may already know that the amount of this space is called *volume*.

Unit 4: Polygons, Prisms, and Cylinders

Lesson 5: Understanding Volume

Get Focused



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Turn to page 246 in your textbook *MathLinks 8*. You can see that Bruce has quite a job ahead of him with so many boxes to move into a shipping container.

If the boxes are cube-shaped, the boxes can be loaded easily. The boxes can fit next to each other with no wasted space.

From the reading on the textbook page, you find out that Bruce knows how many cube-shaped boxes fit on the bottom layer of the container. From this knowledge about the bottom layer, can Bruce figure out the total number of boxes that will fit in the shipping container? How he is able to figure this out will become clear in this lesson.

Determining how many cube-shaped boxes fit in any prism-shaped container is similar to figuring out the container's **volume**. It is possible—and not so hard—to calculate the volume of a prism from just the area of its **base** and its **height**.

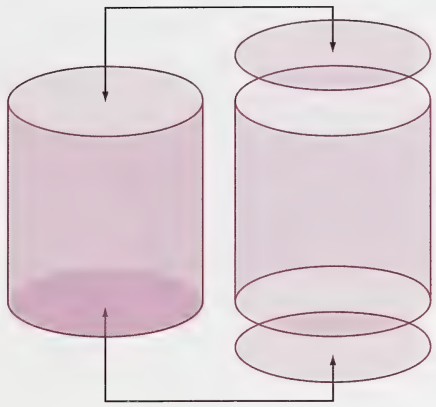
The bases of a cylinder are its two flat circular faces. The bases of a triangular prism are its two triangular faces.

volume: the amount of space an object occupies

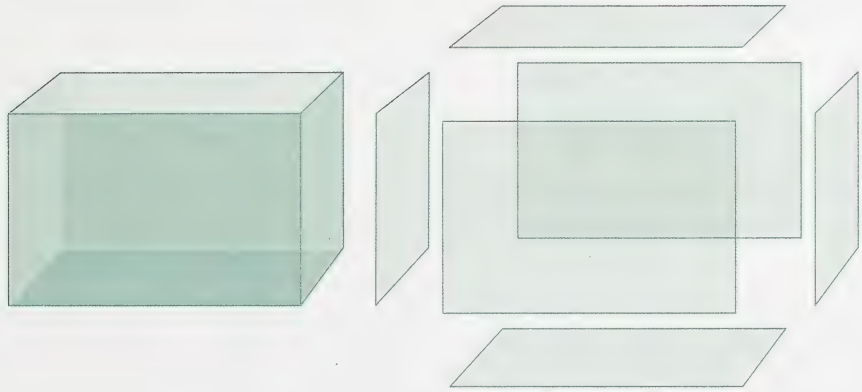
Volume is measured in cubic units, not linear or square units.

base: any face of a prism that is flat and reflects the general shape of the prism

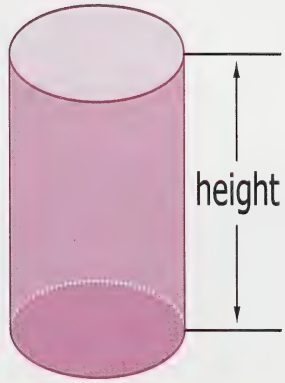
height: the perpendicular distance between opposite faces of a prism or cylinder

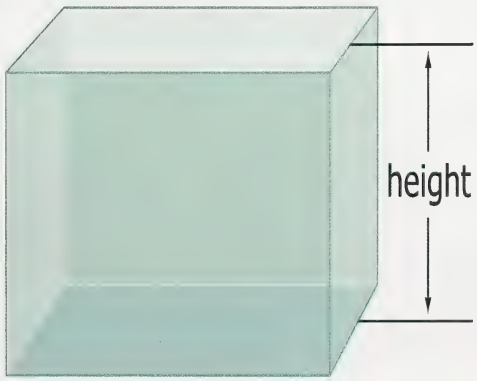


The bases of a rectangular prism are its six rectangular faces.



Even though 3-D objects may have more than one base, the base of a 3-D object usually refers to the lowest base or base at its bottom.





In this lesson you will determine the volume of right rectangular prisms, right triangular prisms, and right cylinders. You will use straightforward ideas to calculate volumes.

You will need a set of approximately 25 cubes of the same size for this lesson. Centimetre cubes, such as cubes having edges of 1 cm, alphabet blocks, or building blocks can be used instead.

This lesson will help you answer the following critical questions:

- What is volume?
- How can you calculate the volume of prisms and cylinders?

Assignments

Your assignment will consist of the following:

- posting to your discussion board
- adding to your Math 8 folder
- completing Unit 4: Lesson 5 Question Set

Explore

Watch and Listen

Go to the Math 8 Multimedia DVD, and open the piece titled "Volume." Select a shape in Volume. Estimate its volume, and then fill the shape with centimetre cubes to determine its volume.

Self-Check

SC 1. Write a definition of *volume* based in your investigation of volume of 3-D objects in the multimedia Volume. Include the words *centimetre cube* or *cubic centimetres* in your definition.

SC 2. The picture on the following page shows, from above, a right rectangular prism made of centimetre blocks.

a. How many blocks make up the top base of the prism?

b. How many blocks make up the bottom base of the prism?

Compare your answers in the Appendix.

In the "Explore the Math" activity presented on pages 246 and 247 of your textbook, you will build various right rectangular prisms from a set of centimetre blocks or a set of other cube-shaped blocks of the same size.

Work with a partner as you do the Try This questions. Your partner may be someone assigned by your teacher, a friend or family member, or someone from your discussion board.

Use your webcam to share each prism you build with someone on your discussion board. Share ideas about the relationship that involves the area of a prism's base, its height, and its volume as you build and study your right rectangular prisms. Remember from a previous lesson that the height of a prism or cylinder is the distance between two bases of the prism or cylinder.

Try This

TT 1. Complete "Explore the Math" questions 1, 2, and 4 of on pages 246 and 247.

My Guide

Q: Why is the area of the base equal to the area of the top face of the prism?

A: Because opposite faces are the same size (and shape) for a right rectangular prism.

Q: If you are using alphabet blocks or building blocks, how do you measure the height of prisms you build?

A: Measure the height by counting the number block edges along a vertical edge of your prism, or count the layers making up the prism. This number gives you how many units high the prism is. For example, if you count five block edges along a vertical edge of your prism, the prism is five units high.

Q: How can you figure out the surface area of the face of the prism you build from blocks?

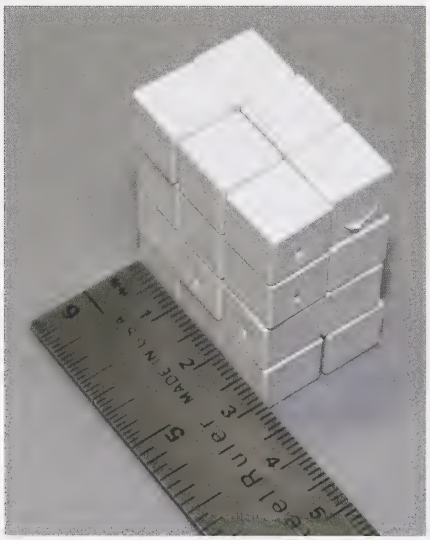
A: The area of a face of a prism is equal to the number of squares making up the face. For example, if you see six squares making up a face, its area is six square units. With centimetre cubes, the area is 6 square centimetres or 6 cm^2.

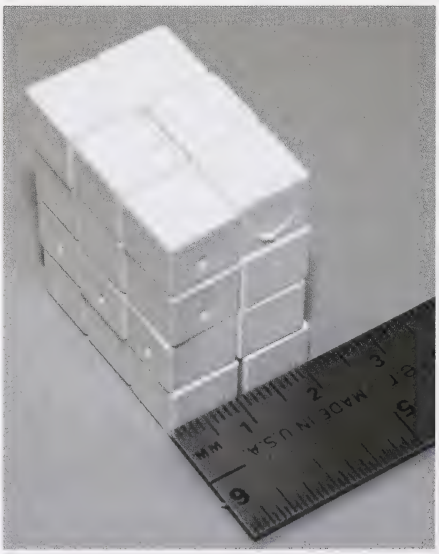
Place a copy of the final version of your answers in your Math 8 course folder.

Connect

Let's now build on what you and your partner discovered during the Explore.

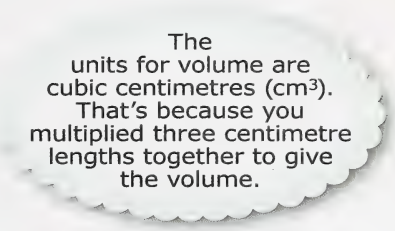
When a prism is built with centimetre cubes, the reading of a centimetre ruler along an edge of the base corresponds to the number of cubes along that edge.





Multiplying the readings gives you the area of the base but also the number of cubes in the lowest layer. In the illustration, the reading is 2 and 3. The product of 2 and 3 is 6. So the area of the base is 6 cm^2, and the number of cubes along the base is 6.

Multiplying the area of the base by the height gives you the total number of cubes in the prism. The number of cubes making up the prism is its volume in cubic centimetres (cm^3).


The units for volume are cubic centimetres (cm3). That's because you multiplied three centimetre lengths together to give the volume.

Self-Check

SC 3. The prism shown in the photo has a height of 4 cm. What is the volume of the prism?

Compare your answers in the Appendix.

There are many prisms that are not made up of individual centimetre cubes. For these prisms, you can use the area of its base and its height to determine the number of centimetre cubes needed to build it. That's also the number that corresponds to its volume.

Read

Read "Example 1: Determine the Volume Using the Base and the Height" on pages 247 and 248 of your textbook to see how the area of a rectangular prism's base and its height are used to determine its volume. Look carefully to see whether this method also works for cylinders.

Self-Check

SC 4. Complete the "Show You Know" question near the top of page 248 of your textbook to determine the volume of a cylinder.

Compare your answers in the Appendix.

Read

Look at the boxes shown in "Example 2: Determine the Volume Using Different Orientations" on page 248 of your textbook. Note how the side measurements are similar, just in different places on the box. Jason's and Mohinder's boxes differ in height, but you can think of these boxes as the same rectangular prism differing only in **orientation**.

orientation: the angular direction and placement of an object

The different positions of an object, formed by translating, rotating, or reflecting the object, are orientations of the object.

Do you think one box would hold a greater number of centimetre cubes than the other? Does one box have a greater volume that the other? How do you know?

Read the example to find out if one of the boxes does have a greater volume than the other.

Self-Check

SC 5. Complete the "Show You Know" question at the top of page 249 of your textbook.

Compare your answers in the Appendix.

Read

Do you remember the important information you learned from this lesson so far? Review "Key Ideas" on page 249 of your textbook for a short summary of the main ideas in the lesson:

- The volume of a 3-D object can be found by multiplying the area of its base and its height.
- The orientation of a 3-D object has no affect on its volume.

Make notes about the key ideas of this lesson for your future study purposes. You might want to add these notes to a new foldable like the one shown on page 244 of your textbook.

Try This

TT 2. Apply the knowledge and skills relating to the volume of 3-D objects by writing your responses to the following. Complete "Communicate the Ideas" questions 1 and 2 on page 250 of your textbook.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 6. Do "Check Your Understanding" questions 3.a), 3.c), 4.a), 4.c), and 5 on pages 250 and 251 of your textbook. For questions 5.a) and 5.b), try to predict the volume of the two prisms before you do the calculations.

My Guide

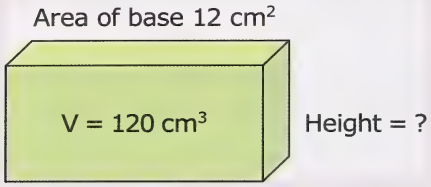
Q: If I know the volume and the area of the base, how do I find the height of the prism?

A: Divide the volume by the area of the base to get the number for height (h).

The explanation for this strategy can come from the formula $V = B \times h$.

If you know the volume and the area of the base (B), then height is the number for h that will make the product of B and h equal to V. Dividing the volume by the area of the base gives you this number for h.

Q: So, how would I apply that in a specific situation? Say this one.



A: You would have the following calculations:

$$V = B \times h$$
$$120 = 12 \times h$$
$$\frac{120}{12} = h$$
$$h = 10$$

In this example, the height would be 10 cm.

SC 7. Do "Check Your Understanding" questions 7.a), 8, 10, 11, and 13 on pages 251 and 252 of your textbook.

Compare your answers in the Appendix.

Extra Practice

How did you do in the Self-Check questions? Did you miss a few answers? More practice would help to make the ideas more concrete in your mind.

If you haven't taken percent yet, then you may not be ready for "Check Your Understanding" question 15.

Turn to pages 250 to 252 in your textbook. Try additional questions from "Check Your Understanding." Then compare your work as you complete it with the answers given on pages 496 and 497 of your textbook. When you feel you are ready, move on to the assignment.

Assignment

Go to the Unit 4 Assignment Booklet, and complete "Unit 4: Lesson 5 Question Set."

Save your answers to this question set. You are to submit your answers according to instructions in one of the lessons that follow.

Going Beyond

To count cubes of structures from their isometric diagram, you may have to stretch your visual skills. See if you are up to the task.

Turn to page 253 of your textbook, and answer "Extend" question 16.

You may want to look at these tips if you need help:

- For question 16.b), find the number of extra blocks that you will need to add to the original structure to make a right rectangular prism. Use centimetre cubes to build the three structures and their associated "completed" prisms, if necessary.
- To envision the "completed" prism for each structure, think of the prism as having three layers, where each layer has a rectangular shape just large enough to contain the base of the original structure.
- To find the total number of cubes in the completed prism, you can do either of the following:
    - Multiply the number of blocks in the completed base by the height of the "completed" prism.
    - Add the number of cubes in the original structure and the number you found you needed to make the "completed" prism.
- To find the volume of the completed prism, multiply the volume of one cube, which is 8 in cubic centimetres, by the total number of cubes in the "completed" prism.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you determined the volume of right rectangular prisms, right triangular prisms, and right cylinders. You used simple formulas to calculate volumes.

The volume of prisms and cylinders can be found by determining the number of cubes needed to fill the object. From this, you discovered that the volume of prisms and cylinders can be found by multiplying the area of the object's base by its height.

Go to the Unit 4 Summary and begin working on part 2 of your Unit Problem.

In the next lesson you will develop and use formulas to calculate volume from the dimensions of 3-D objects.

Unit 4: Polygons, Prisms, and Cylinders

Lesson 6: Volume of a Prism

Get Focused

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Many products that you select from store shelves are packaged in paper and cardboard. When placed in recycle bins, such packaging is collected, pressed into bales, and transported to recycling plants. Then the material is finally recycled into new products. Recycling materials reduces the impact on the environment.

By using recycle bins, people do an important part in helping to care for the environment. As you read on page 254 of the textbook, a variety of materials—paper, aluminum, plastic, and glass—can be recycled.

The reading from the textbook leaves you with a question about a bin shaped like a right rectangular prism. In order to know how much paper was recycled when the bin is filled with paper, you would need to find the volume of the bin's prism shape.

In this lesson you will develop and use formulas for determining the volume of rectangular prisms and triangular prisms.

This lesson will help you answer the following critical question: How can you calculate the volume of prisms using a formula?

Assignments

Your assignment will consist of the following:

- posting to your discussion board
- adding to your Math 8 folder
- completing Unit 4: Lesson 6 Question Set

Explore

In the previous lesson you worked out the volume of prisms from the area of the base of the prism and its height. In the "Explore the Math" activity presented on pages 254 and 255 of your textbook, you will not be provided with the base area. Instead, you will use linear measurements to calculate the volume of prisms.

Discuss and Share

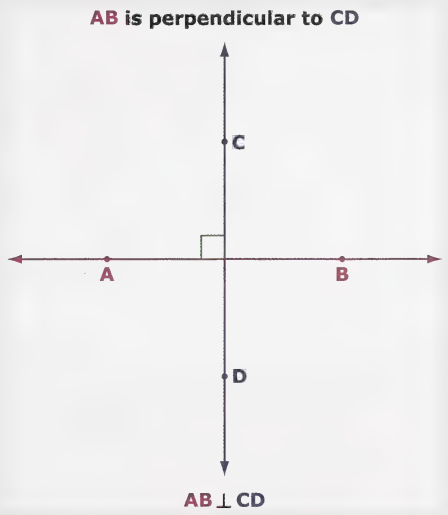
Work with a partner so that you can talk about what you are doing as you complete the Self-Check questions. Your partner may be someone assigned by your teacher, a friend or family member, or someone from your discussion board.

After completing the Self-Check questions, discuss methods you can use to find the volume of any right rectangular prism or right triangular prism, especially when the area of the base is not known directly. Share examples you use to test your methods for finding volume.

perpendicular: being at right angles (90°) to a given line

Two lines are perpendicular (to each other) if they intersect at right angles (90°).

Example:



Self-Check

SC 1. Complete "Explore the Math" questions 1, 2, and 3 on page 254.

In the following answers, you will be introduced to the term **perpendicular**.

Compare your answers in the Appendix.

Based on your investigation of specific triangles, can you describe your methods for finding volume that work for any right rectangular or triangular prism? Expressing your method using a formula is the idea behind the next Try This question.

My Guide

Q: How can you determine the volume of a prism if you know the area of its base?

A: You multiply the area of the base by the height of the prism.

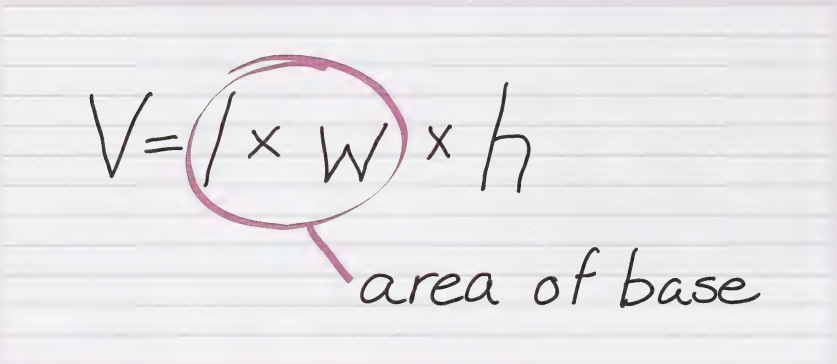
Q: For a rectangular prism, how can you find the area of the base from the dimensions of the prism?

A: One of the prism dimensions is the height of the prism. The other two dimensions are the length and width of the base. The area of the base is found by finding the product of these other two dimensions of the prism.

area of base = length x width

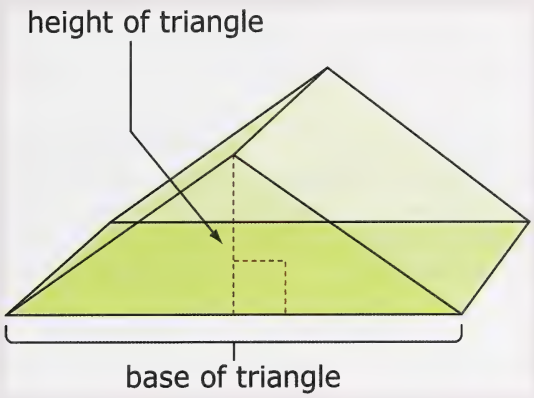
Q: How can you write the product of the area of the base and the height of the prism as a product of the dimension of the prism?

A: $V = l \times w \times h$



Q: How can you find the area of the base of a right triangular prism?

A: You find the area of the base as you would for any triangle. You multiply its height by the length of its base as a triangle and divide by 2. Be careful not to confuse the base of this triangle with the base of the triangular prism.



Q: To get the volume of the prism, by what do you have to multiply the area of the triangular base?

A: You have to multiply this area by the height of the prism.

Try This

TT 1. Complete "Explore the Math" question 4 on page 255.

Place a copy of the final version of your answer to question 4 in your Math 8 course folder.

Connect

Let's now build on what you and your partner discovered during the Explore.

Read

Read "Example 1: Use a Formula to Determine the Volume of a Right Rectangular Prism" on page 255. The example demonstrates how the volume of a prism can be determined by applying a formula. See how the formula for a rectangular prism can be simplified when it is used to find the volume of a cube.

Self-Check

SC 2. Complete the "Show You Know" questions at the bottom of page 255 of your textbook to determine the volume of a rectangular prism by applying a formula.

Compare your answers in the Appendix.

Read

"Example 2: Use a Formula to Determine the Volume of a Right Triangular Prism" on page 256 of your textbook shows how to use a formula to determine the volume of a rectangular prism. This formula involves division by 2. Try to remember why division by 2 is part of this formula.

Self-Check

SC 3. Complete the "Show You Know" question in the middle of page 256.

Compare your answers in the Appendix.

Read

"Example 3: Use Volume to Solve a Problem" on page 256 of your textbook shows how you can solve problems related to volume. This example will show you how to determine how much empty space is at the top of a partially full cereal box.

Self-Check

SC 4. Complete the "Show You Know" question in the middle of page 257 of your textbook.

Compare your answers in the Appendix.

Read

Do you remember the important information you learned from this lesson so far? Read "Key Ideas" on page 257 of your textbook for a short summary of the main ideas in the lesson. Pay close attention to the formulas to determine the volume of right rectangular prisms and right triangular prisms. If you still have questions or are unclear about how to use a formula, be sure to contact your teacher for additional support.

Make notes about this summary for your future study purposes. You might want to add these notes to a foldable like the new one shown on page 244 of your textbook.

Try This

TT 2. Apply the knowledge and skills you have developed relating to the volume of prisms. Complete "Communicate the Ideas" questions 1, 2, and 3 on page 258 of your textbook.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 5. Do "Check Your Understanding" questions 4, 5.a), 5.c), 6.c), 7.a), 7.b), 8.a), 8.b), 9.a), 9.c), 10.a), and 10.c) on pages 258 and 259 of your textbook.

Compare your answers in the Appendix.

SC 6. Do "Apply" questions 11, 12, 13, 14, and 15 on page 260.

Compare your answers in the Appendix.

Extra Practice

If you're still feeling a bit challenged by the ideas presented in the lesson, more practice might help you become comfortable with the ideas.

For extra practice, you may complete additional "Check Your Understanding" questions on pages 258 to 261. Then compare your work after you complete each additional question with the answers given on page 496 of your textbook.

When you feel confident with your skills in solving volume-related questions, complete the lesson assignment.

Assignment

Go to the Unit 4 Assignment Booklet, and complete "Unit 4: Lesson 6 Question Set."

Submit your answers to the Lessons 5 and 6 Question Sets to your teacher for evaluation.

Going Beyond

Have you noticed that the level of water changes far more quickly in a sink compared to a bathtub? The length and width of the tub is far greater than that of a small sink, which accounts for the difference in the rate at which the water levels change.

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Knowing how water levels change when water is poured into a container helps you understand what happens when water is drained from a container. They are reverse processes, aren't they?

How does the water level change in a container when a certain volume of water is drained from the container? By using a rectangular prism-shaped tank, this relation can be analyzed using skills from this lesson. Such an analysis is involved in the following question.

Turn to page 261 of your textbook, and solve "Extend" question 24.

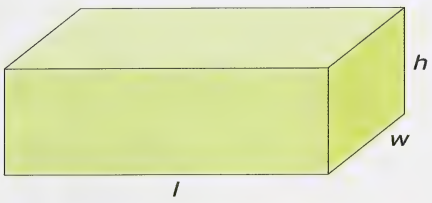
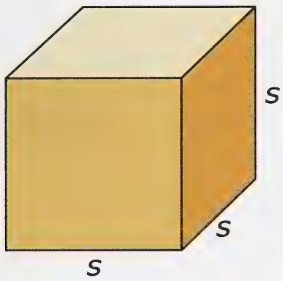
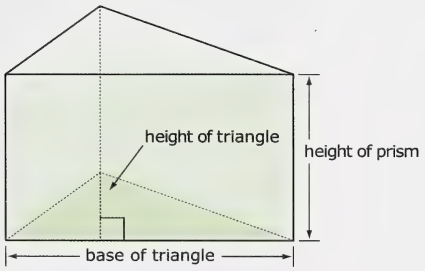
Compare your answers in the Appendix.

Lesson Summary

There are so many containers around you that have the shape of a right prism. You can use the formulas presented in this lesson to figure out how much a right rectangular prism-shaped cereal box can hold or the amount of chocolate candy that fits in a right triangular prism. The calculations involve the concept of volume.

In this lesson you discovered how to use formulas to determine the volume of right rectangular prisms and right triangular prisms.

You can calculate the volume of prisms using several formulas:

For a right rectangular prism:  	$V = l \times w \times h$
For a cube:  	$V = s \times s \times s$ or $V = s^3$
For a right triangular prism:  	V = (base length of triangle × height of triangle ÷ 2) × height of prism

Remember to continue your work on your Unit Problem. Refer to the Unit 4 Summary, and work on part 2. Remember, your teacher is also available to answer questions you may have concerning the unit problem. Be sure to make use of this help as soon as you need it.

In the next lesson you will build upon your knowledge of calculating volume by developing and using formulas to calculate volume of right cylinders.

Unit 4: Polygons, Prisms, and Cylinders

Lesson 7: Volume of a Cylinder

Get Focused

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Towns and cities often pump water into storage tanks to ensure there is enough water on hand for the ongoing demands of residents.

Turn to page 262 in your textbook *MathLinks 8*. A cylinder-shaped water storage tank is described. The reading leaves you to make a prediction about a town's water supply. You will be able to make an accurate prediction when you know how to determine the volume of a cylinder.

In this lesson you will determine the volume of a cylinder using a formula that you set out to develop.

This lesson will help you answer the following critical question: How can you calculate the volume of cylinders using a formula?

In this lesson you will need 1-cm grid paper; a variety of used tin cans; a 250-mL measuring cup; sand, rice, or water; and a ruler. You can find "1-cm grid paper" on your Math 8 Multimedia DVD.

Assignments

Your assignment will consist of the following:

- posting to your discussion board
- adding to your Math 8 folder
- completing Unit 4: Lesson 7 Question Set

Explore

In the previous lesson you developed formulas for the volume of prisms. In the "Explore the Math" activity presented on page 262 of your textbook, you will develop a formula for the volume of a right cylinder.

In Lesson 6 you used solid cubes to explore the relation between the area of the base of a right rectangular prism and the volume of the prism.

You will continue to explore concepts in this lesson using materials. But instead of solid cubes, which don't fit nicely in round containers, you will use materials such as sand or water.

In science you have likely done some investigation of volume. You may have used a strategy that involves measuring the **capacity** of containers in order to determine their volume. That strategy—involving measurement of capacity—may help you in this lesson.

capacity: the greatest volume of fluid that a container such as a tank, flask, or a cup can hold

Collect the items listed in the margin note "Materials" on page 262 of your textbook.

Discuss and Share

Work with a partner. Discuss various methods you can use to determine the volume of cylinders using a ruler, the 1-cm grid paper, a measuring cup, and materials that can be poured into the cylinder-shaped container. Your partner is there to bounce ideas around with and help you to think about the Explore activity.

As you complete the activity, share digital pictures of your containers and the measurement data you collect for the containers with others. Use the pictures and data to verify each other's results.

My Guide

Q: I filled a pop can with water to determine its capacity. My partner then poured the water from the pop can into a measuring cup. This showed that that the pop can contained about 370 mL of water. What should I say was the volume of the pop can? Why?

A: The capacity of the pop can is 370 mL. Since 1 mL = 1 cm^3, you can say that the volume of the pop can was 370 cm^3.

Q: How could I use the information on pages 246 to 249 of the textbook to help me calculate the volume of this cylinder?

A: According to that textbook lesson, you calculate the area of the base and multiply the base by the height. You can measure the area of the base of a cylinder by tracing the circle shape of the base on centimetre grid paper. The number of squares—and fractions of squares—contained in the circle will correspond to the area of the base in cubic centimetres.

Q: Should calculation of volume based on area of the base and height agree with the measured capacity of the container?

A: The calculations should supposedly agree for any cylinder that has thin walls. Some tin cans, however, may only be partially cylindrical. For example, the curved walls of many pop cans slope to a smaller radius at the top and bottom ends.

Also, the bottom of a pop can is not flat but curves inward. Such features can throw off the measured volume of the tin can. Try to use tin cans that are very close to being cylindrically shaped.

Q: How many decimal places should I use in my calculation?

A: Keep as many decimal places as you can on your calculator. Only at the end should you round decimal values to a reasonable number. Keeping as many decimal places as possible in the steps of your calculations will help you get a more reliable final answer. You may round your final calculated answer to the nearest whole number of cubic centimetres.

Try This

TT 1. Complete "Explore the Math" questions 1, 2, 3, and 4 on page 262. In your answers, describe the tin cans you used along with measurements you took from them. For question 2, you may also use the idea of capacity to measure—rather than calculate—the volume of the cylindrical can.

Revise any of your responses, as needed, after discussing TT 1 with your partner.

Place a copy of the final version of your answers in your Math 8 course folder.

Connect

Let's now build on what you and your partner discovered, with the help of others on your discussion board, during the Explore.

Read

Read "Example 1: Determine the Volume of a Cylinder Given the Radius" on page 263 of your textbook to see how the volume of a cylinder can be determined from just its radius and height. Notice where and how rounding of decimals takes place in the solution of the example.

Self-Check

SC 1. Complete the "Show You Know" questions at the bottom of page 263 of your textbook to determine the volume of a cylinder by applying a formula used in "Example 1."

Compare your answers in the Appendix.

Read

A cylinder does not need to have a great height. Even a round penny is a cylinder. The next example deals with a hockey puck, which, like a penny, is a rather flat cylinder.

Work through "Example 2: Determine the Volume of a Cylinder Given the Diameter" on page 264 of your textbook. Pay close attention to how the diameter value is used in the calculations.

Self-Check

SC 2. Complete the "Show You Know" question in the middle of page 264 of your textbook.

Compare your answers in the Appendix.

Read

Read "Key Ideas" on page 264 of your textbook for a short summary of the main ideas in this lesson. There are formulas for the area of the base of a cylinder and for its volume you will need to remember. Make notes about this summary for your future study purposes. Add these notes to a foldable like the one you created for this part of the unit that is shown on page 244 of your textbook.

Try This

TT 2. Now here's a chance to apply your knowledge and skills relating to the volume of cylinders. Complete "Communicate the Ideas" questions 1, 2, and 3 on page 265 of your textbook.

Include these with other problems you have completed in this unit and placed in your Math 8 course folder.

Self-Check

SC 3. Complete "Check Your Understanding" questions 4, 5.a), 5.c), 6.a), 6.b), 7.a), and 7.c) on pages 265 and 266 of your textbook.

SC 4. Complete "Apply" questions 8, 10, and 12 on page 266 of your textbook.

Compare your answers in the Appendix.

Extra Practice

You've already done a lot of work in this lesson so far, but are you still feeling challenged by the methods to calculate the volume of cylinders? Or do you think "practice makes perfect"?

If so, you may choose to complete additional questions from "Check Your Understanding" on textbook pages 265 to 267 for additional opportunities to work at calculating volume of cylinders. After each question, check your answers on pages 496 and 497 of your textbook.

Assignment

Go to the Unit 4 Assignment Booklet, and complete "Unit 4: Lesson 7 Question Set."

Save your answers to this question set. You are to submit your answers according to directions in the lesson that follows.

Going Beyond

In this lesson you learned to use a formula to determine the volume of a cylinder. For some real-world applications, this formula has to be modified. Are you up to the task? Then continue reading.

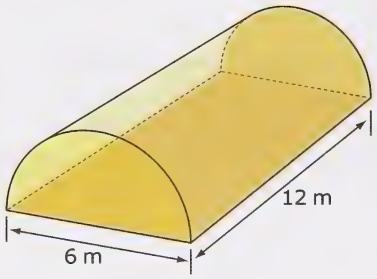
Look at the following structures.

Left to right: © Richard Thornton/shutterstock; © Richard Thornton/shutterstock

You can see that each picture shows a structure having a semi-circular arching roof that curves downward to form walls. These structures, known as Quonset huts, were designed so they could be assembled quickly and easily from manufactured parts.

1. Adapt the formula $V = (\pi \times r^2) \times h$ so that it can be used to find the volume of Quonset huts from the values for r and h. Explain your new formula.

2. At the local hardware store, there is only one standard size of Quonset hut available for purchase. The measurements for this Quonset hut are as follows:

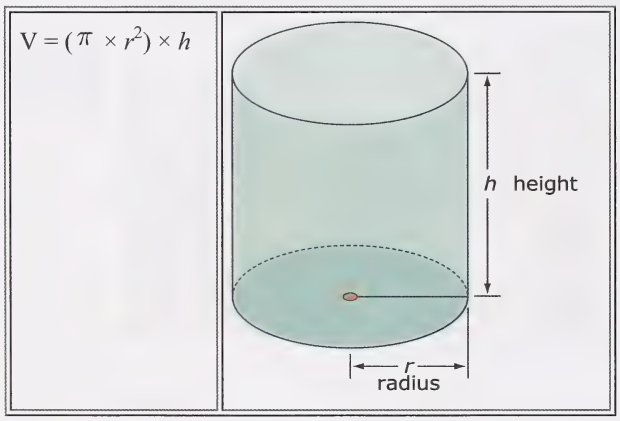


Determine the volume of this Quonset hut using the values given in the diagram and using the modified formula you created in part 1 of this question. Show your calculations.

Compare your answers in the Appendix.

Lesson Summary

Congratulations on the completion of this lesson, in which you developed and applied formulas for the volume of cylinders.
You can calculate the volume of cylinders using the formula:



Continue your work on your Unit Problem. Refer to the Unit 4 Summary, and work on part 2 of the Unit Problem. Keep in mind that you can talk to your teacher if you have questions about how you are progressing in your project.

In the next lesson you will be looking at solving problems involving the volume of right rectangular prisms, right triangular prisms, and right cylinders.

Lesson 8: Solving Problems Involving Prisms and Cylinders

Get Focused

You may think of coins simply as tokens that allow you buy merchandise. But some coins are themselves packaged and sold as merchandise. Besides the coins shoppers use every day, the Royal Canadian Mint also issues collector coins. These coins are valued by coin collectors around the world.

This $100 gold coin celebrates the tenth anniversary of the creation of Nunavut.

© Coin image © 2009 Royal Canadian Mint – All Rights Reserved/Image de pièce© 2009 Monnaie royale canadienne – Tous droits réservés

© 2009 Jupiterimages Corporation

Coin collectors have looked forward to the Royal Canadian Mint's special $100 gold coin that commemorates the tenth anniversary of Nunavut in 2009. This coin honours the creation of Canada's newest territory—Nunavut. On one side of the coin is a brilliant illustration by Canadian Inuit artist Andrew Qappik, who drew a drum-dance scene and the faces of an Inuit Elder, adult, and youth. In this way, he celebrated Nunavut's cultural heritage and its future.

Coin collectors not only take an interest in the artistic design of coins but also the technical characteristics of coins. The toonie is especially interesting as a bi-metallic coin. The toonie has an outer diameter of 28 mm, and its inner core has a diameter of 17 mm. The inner ring is mainly copper, and its outer ring is almost completely nickel. The coin is 1.8 mm thick.

Suppose you wanted to determine the volume of metal needed to make the coin or just the volume of nickel making up the ring. From the technical information, you will be able to figure out this information. But this takes some problem-solving skill with 3-D objects.

In this lesson you will gain experience solving problems that involve right rectangular prisms, right triangular prisms, and cylinders.

This lesson will help you answer the following critical question: Why is the volume of a prism- or cylinder-shaped object important?

In the Explore you are presented with a problem to solve. Depending on the strategies you use to solve the problem, you may want to use centimetre cubes or building blocks, 1-cm grid paper, and possibly some other materials that you have on hand.

Assignments

Your assignment will consist of the following:

- posting to your discussion board
- adding to your Math 8 folder
- completing Unit 4: Lesson 8 Question Set

Explore

Imagine Danielle's situation that is pictured and described on page 268 of your textbook. Her problem involves volume and the shape of right rectangular prisms. You will solve Danielle's problem in Self-Check question 1.

This problem is likely somewhat different from any you have solved before. The problem provides you with information you do know and asks you to apply that knowledge in a new way. This is the essence of problem solving—looking at challenges with ideas you already have and looking at ways of connecting them. Working with others to help you solve the problem is a good strategy to help you discuss what you know and how potential solutions might work themselves out.

Discuss and Share

Group work can be very effective in meeting challenges that would seem too hard to solve alone. Work with two, three, or even four partners as you complete the Explore questions based on Danielle's situation. Your partners may include friends, family members, or other students from your discussion board. Communicate with instant messaging or video chat as you work

through the questions so you can pose questions and reply as you work. You may also want to share digital pictures or short video clips to show diagrams or models to the others.

Self-Check

SC 1. Determine the volume of the little car box and the crate that are described on page 268. Then, based on your calculations, estimate the number of car boxes that Danielle could fit in the crate.

Compare your answers in the Appendix.

Try This

TT 1. You may use the simulation Filling a Crate with Boxes to model Danielle's problem and observe one way of stacking boxes into the crate. Go to the Math 8 Multimedia DVD, and open the piece titled "Filling a Crate."

a. Show the maximum number of boxes that Danielle should be able to fit into the box. You may present your method of packing using a diagram for each layer or region of the crate. Or, if you are using the simulation Filling a Crate with Boxes, you have the option of using screen shots to show the stages of your solution.

b. Indicate the number of boxes Danielle can pack into the crate using your method of packing the crate.

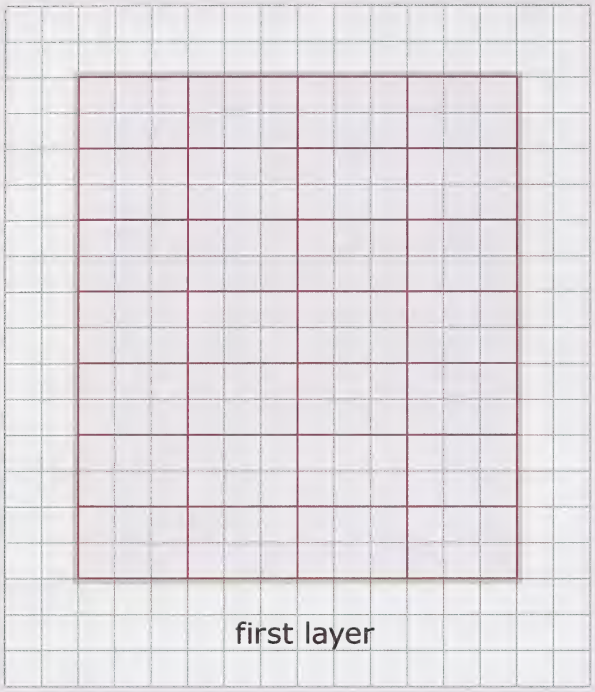
My Guide

Q: How can you use centimetre grid paper to help you?

A: The grid paper allows you to make a scale drawing of each layer from the top view to make sure space is used well.

Q: How can you decide which way to put in the boxes?

A: Here's one way to build the first layer. Notice how divisibility is used. The 30-cm edge of the toy boxes are placed in the direction of the 120-edge since 120 is divisible by 30 but 140 is not. The number 140 is divisible by 20 but not 50, so the 20-cm edge of the toy boxes should placed in the direction of the 140-cm edge. From the top, the bottom layer looks like this.



This layer contains 28 car boxes. See if you can build this layer of boxes using the simulation Filling a Crate with Boxes.

Q: That leaves 60 cm to the ceiling of the crate. If you make another layer this way, you would have 10 cm left to the ceiling but another 28 boxes left to be packed. Should you try to make two layers, each 30-cm high? That would take you to the ceiling.

A: Yes, you could try that using the simulation Filling a Crate with Boxes. The bottom face of the toy boxes would be 50-cm long and 20-cm wide. You will have to decide which way you should orient these faces to get the greatest number of boxes on these layers. You will notice that there will be a right rectangular prism-shaped space left over on one side—above the first layer—that you can use to place five additional boxes.

Place a copy of the final version of your solution in your Math 8 course folder.

Try This

TT 2. In SC 1 you used the volume of the car box and the crate to estimate the number of car boxes that Danielle could fit in the crate. Complete "Explore the Math" question 5 on page 269 to reflect on your estimate.

TT 3. In Danielle's problem the remote control car boxes were all right rectangular prisms. How would the problem have been easier to solve if the car boxes had been cube shaped rather than oblong?

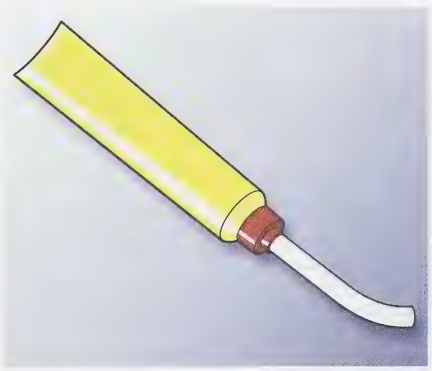
Place a copy of the final version of your TT 2 and TT 3 answers in your Math 8 course folder.

Connect

In the Explore activity, visualization—that is, thinking in pictures and images—was used to help you understand how many rectangular prisms fit into one rectangular prism. Let's now build on your use of visualization as a tool in problem solving.

Read

As you go over "Example 1: Solve a Problem Involving Right Triangular Prisms" on pages 269 and 270 of your textbook, notice how drawings allow you to visualize the situation. Drawings also provide you with the opportunity to show how prisms can fit together and test if the basis for your mathematical calculations is based on a logical idea.



You may want to think of a tube of toothpaste as you read "Example 2: Solve a Problem Involving Cylinders" on textbook page 270.

When you squeeze a tube of toothpaste and force the contents through the hole at the end of the tube, the material can come out almost like a cylinder. This cylinder of toothpaste has a radius much smaller than the original radius of the tube. You can imagine that squeezing out all the toothpaste this way would lead to a very long cylinder of toothpaste—much longer than the tube. You've actually moved the toothpaste from one cylinder through the nozzle into another cylinder having a much smaller radius.

In "Example 2," the new cylinder is to be made to have a smaller radius than before. Based on your experience with a toothpaste tube, you could expect the thinner cylinder to be longer. See if the formula does show the new cylinder to be longer.

Self-Check

SC 2. Complete the "Show You Know" question at the top of page 271 of your textbook to determine the length of a thicker cylinder.

Compare your answers in the Appendix.

Read

"Example 3: Solve a Problem Involving Right Prisms and Cylinders" on page 271 of your textbook shows how to use the formulas to determine the volume of a rectangular prism and a cylinder to solve a multi-step problem. In the problem how did you know that there was more than one calculation that needed to be solved for?

Self-Check

SC 3. Complete the "Show You Know" question at the top of page 271 of your textbook to demonstrate that you understand how to solve a multi-volume problem.

Compare your answers in the Appendix.

Read

Review "Key Ideas" on page 272 of your textbook for a short summary of the lesson. Make notes about this summary for your future study purposes or to use while completing the rest of this lesson. You might want to add these notes to a foldable like the new one shown on page 244 of your textbook.

Try This

TT 4. Apply your problem-solving skills by writing your responses to the following. Complete "Communicate the Ideas" questions 1 and 2 on page 272 of your textbook. Be as thorough with your explanation as possible because being able to explain how to solve a problem is an important step in being able to solve it.

Discuss and Share

Post your answers on the discussion board for others to react to. Also read one or two other posted answers and respond to those answers. Revise your answer using feedback and other postings.

Place a copy of your revised answers in your Math 8 course folder.

Self-Check

SC 4. Complete "Check Your Understanding" questions 3, 4, and 7 on page 273 of your textbook. Remember that you can look back to the examples that you read through earlier in the lesson to help you understand what process you might need to use to solve the problems.

SC 5. Complete "Check Your Understanding" questions 8, 14, and 17 on pages 273, 274, and 275 of your textbook. If you are feeling stuck in what to do for the problem, remember to set out a plan for how to solve the problem like you did in TT 4. Knowing what steps you will need to take and in what order can help solve the problem.

Compare your answers in the Appendix.

Extra Practice

Still feeling challenged trying to solve the volume problems in the lesson? More practice might help you become comfortable and confident with problem solving related to prisms and cylinders.

Turn to pages 273 to 275 in your textbook. For extra practice, you may want to complete additional questions from "Check Your Understanding." After each question, compare your work with the answers given on pages 497 of your textbook. Keep in mind that the textbook provides brief answers. If you are still feeling stumped, give your teacher a call for some extra help.

When you are ready, complete the Unit 4: Lesson 8 Question Set.

Assignment

Go to the Unit 4 Assignment Booklet, and complete "Unit 4: Lesson 8 Question Set."

Submit your solutions to the Lessons 7 and 8 Question Sets to your teacher for evaluation.

Going Beyond

What happens when an object is made up of a combination of shapes? Challenge yourself to see if you can use what you know about calculating volumes to solve a real-world situation.

Turn to page 275 of your textbook and solve "Extend" question 21.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you developed and used a variety of strategies to solve problems relating to the volume of prisms and cylinders. Often formulas helped in problem solving, but knowing which formula to apply and at what step of the problem to apply it was very important in solving the problem successfully. You also found that diagrams helped you to visualize the situations in the problems to better understand what was being asked.

You have seen many ways in which the volume of a prism- or cylinder-shaped object was important. Think about the value of the contents in a container. The greater the volume of a container, the more the value it could hold. That's why the price of a 750-mL container of juice is greater than a 320-mL container of juice. The construction cost of some objects was connected closely to volume. Think of a culvert. The greater its volume, the more concrete was needed to make it. The volume of objects is also related to how many objects can be stacked in a certain space. You saw that the volume of toy car boxes is related to the number that can be stacked inside a given crate.

So suppose you are responding to this question: Why is the volume of a prism- or cylinder-shaped object important? Thinking about the value, the cost, and the stability when stacked can be helpful clues.

The Unit 4 Summary follows this lesson. You should already be working in the Unit 4 Summary to complete part 2 of the Unit Problem. Once you have completed your Unit Problem, finish reading the rest of the Unit 4 Summary and answering all of the questions.

Unit 4: Polygons, Prisms, and Cylinders

Unit 4 Summary

Getting Started

You know that it would be hard to organize the belongings you have around your home without storage containers. Tools go into the toolbox: needles, thread, and thimbles go into the sewing box; cookies into the cookie jars, spaghetti into pasta dispensers; chess pieces into the chess box; pens into the pen container; and so on.

left: © Sergieiev/shutterstock; centre: © creatOR76/shutterstock; right: © soMeth./shutterstock

Some containers you use around the home may be fairly plain. Wouldn't it be great to have a few containers that were attractive, practical, and decoratively labelled? There must be other people who would like specially designed containers such as these.

Unit Problem

© Paul Marcus/shutterstock

Imagine being the product designer for a specialty store that features storage containers for people who want appealing, practical, and decoratively labelled solutions to their clutter issues. You will apply your knowledge about views, surface area, volume, and orientation of 3-D objects as you tackle this Unit Problem.

The store owner wants you to design a new container that is appealing, practical, and decoratively labelled. In addition, you are asked to design the display case to show about 30 to 40 of the containers as a featured product to store customers. The owner wants to spark interest by having a display case created in a different kind of shape from the containers inside. For example, if the display case is a triangular prism, then the manager would like the container shaped like a cylinder or a rectangular prism.

As the designer, the choice of materials the display case and container are made of is completely up to you. They can be made of a combination of materials or just one type of material. You also do not have to worry about how the container or display case will be assembled. The manufacturer will figure this out from your design drawings.

Look over both parts of this Unit Problem, so that you have a good understanding of how the whole problem will fit together. The choices you make to complete part 1 can make the completion of part 2 more complicated than the problem needs to be. For example, if you design a container or display case that has a lopsided or bumpy shape, finding its surface area or volume may be too difficult.

If you find yourself struggling with any of the Try This items, contact your teacher for assistance as soon as possible. Asking for help early on may prevent unnecessary uncertainty or from your having to redo components of the project.

As you complete the Unit Problem, you may want refer to a generic Project Rubric for Students. You can use this rubric to guide the quality of your work.

Project Rubric for Students

Score/ Level	Holistic Descriptor
5 (Standard of Excellence)	☐ I have applied/developed **thorough** strategies and mathematical processes making **significant** comparisons/connections that demonstrate a **comprehensive** understanding of how to develop a complete solution. ☐ My procedures are **efficient and effective** and may contain a **minor mathematical error** that does not affect understanding. ☐ I've used **significant** mathematical language to explain my understanding and provided **in-depth** support for my conclusion.
4 (Above Acceptable)	☐ I have applied/developed **thorough** strategies and mathematical processes for making **reasonable** comparisons/connections that demonstrate a **clear** understanding. ☐ My procedures are **reasonable** and may contain a **minor mathematical error** that may hinder the understanding in one part of a complete solution. ☐ I've used **appropriate** mathematical language to explain my understanding and provided **clear** support for my conclusion.

3 (Meets Acceptable)	☐ I have applied/developed **relevant** strategies and mathematical processes making **some** comparisons/connections that demonstrate a **basic** understanding. ☐ My procedures are **basic** and may contain a **major error or omission**. ☐ I've used **common** language to explain my understanding and provided **minimal** support for my conclusion.
2 (Below Acceptable)	☐ I have applied/developed **some relevant** mathematical process(es) making **minimal** comparisons/connections that lead to a **partial solution**. ☐ My procedures are **basic** and may contain **several major mathematical errors**. ☐ My communication is **weak**.
1 (Beginning)	☐ I have applied/developed an **initial start** that may be **partially correct** or could have led to a correct solution. ☐ My communication is **weak or absent**.

Adapted from Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), Master Project Rubric 1. Reproduced by permission.

Part 1

You will complete part 1 of the Unit Problem as you work on Lessons 1, 2, 3, and 4. Complete the following steps.

TT 1. Make a visual representation of your design for the container by making the following:

- a free-hand or electronic drawing of the container that shows its dimensions and where any decorative labels will be placed
- a sketch of the top, side, and front views of the container
- a net of the container

Include any special features, such as the decorative labelling on the surface of the container.

TT 2. Make a visual representation of your design for the display case by making the following:

- a free-hand or electronic drawing of the container that shows its dimensions
- a sketch of the top, side, and front view of the display case
- how the containers are to be stacked inside the display case

Include any special features such as open sides or top.

TT 3.

a. Calculate the surface area and also how much of the surface area of your container will be used for decorative labelling. Show the formulas you used and your calculations.

b. Describe or show the orientation of the container inside the display case. Explain why this orientation is used to place the container in the display case.

Once you have completed Lesson 4, go on to part 2.

Part 2

You will complete part 2 of the Unit Problem as you work on Lessons 5, 6, 7, and 8.

TT 4. Determine the combined volume of all of the containers in the display case. Make sure to show the formulas you used and your calculations.

TT 5. Determine the volume of the display case. Make sure to show the formulas you used and your calculations.

TT 6. Determine the proportion of the space inside the display case that is occupied by the containers. Show the calculations you used to solve for this. You may express your answer as a decimal fraction, a common fraction, or a percentage. Explain why some space would be left unoccupied by the containers.

TT 7. Recommend a use for your container. Provide reasons for your recommendation based on physical characteristics of the container. Show any mathematical reasoning you used to support your recommendations.

My Guide

Q: What is meant by *mathematical reasoning*?

A: Mathematical reasoning is the logical thinking you use in applying or developing mathematical concepts, strategies, or processes. You use mathematical reasoning when solving math problems or when using mathematical concepts, strategies, or processes to make a claim.

For example, suppose you claimed that a cylindrical container having a height of 8 cm and a radius of 3 cm would be suitable as a lemonade glass. In making that claim you could show, using the formula $V = \pi r^2 h$, that this container had a capacity close to a glass of water, which is about 237 cm^3. Defending this claim in this way involves mathematical reasoning.

TT 8. Design an ad—either for print or digital media—that promotes the new containers. Your ad should provide reasons why your container is so desirable for the use you recommend for the container.

Before you hand in your project for evaluation, go over the checklist and take a moment to ensure that your Unit Problem is complete.

Checklist

Before you submit your Unit Problem to your teacher, use this checklist to make sure you have completed all of the questions.

Part 1

TT 1: I have drawn a visual representation of a container.
TT 2: I have drawn a visual representation of a display case.
TT 3: I have calculated the surface area and the area used for labelling.

Part 2

TT 4: I have determined the volume of containers and provided the formulas I used and my calculations.
TT 5: I have determined the volume of the display case and provided the formulas I used and my calculations.
TT 6: I have determined the proportion of the space inside the display case that is occupied by the container.
TT 7: I have provided a description of the use for the container.
TT 8: I have created an ad promoting the container.

Place a copy of your responses to the Try This questions in your Math 8 course folder.

Unit Summary

Congratulations on finishing Unit 4! In this unit you studied right prisms, right cylinders, and other 3-D objects. You learned to represent 3-D objects in a 2-D form as nets. You also developed and applied strategies to determine the surface area and volumes of right prisms and of right cylinders. Some of your strategies were based on the use of formulas for surface area and volume.

surface area of a right prism:	SA = (area of front and back) + (area of the top and bottom) + (area of ends)
surface area of a cylinder:	$SA = 2 \times (\pi \times r^2) + (\pi \times d \times h)$
volume of a right rectangular prism:	$V = l \text{ x } w \text{ x } h$
volume of a cube:	$V = s \text{ x } s \text{ x } s$ or $V = s^3$
volume of a right triangular prism:	V = (base length of triangle × height of triangle ÷ 2) × height of prism

You used your knowledge of 3-D objects to act as a design for a new container product and the display case to showcase it in.

Unit Review

This unit was based on two chapters in your textbook—"Chapter 5: Surface Area" on pages 160 to 193 of your textbook and "Chapter 7: Volume" on pages 242 to 281. Your unit review will be based on the two related textbook chapter reviews.

Self-Check

SC 1. Turn to "Chapter 5 Review" on pages 188 and 189. Complete questions 1, 2, 3, 4, 5, and 6. Then answer at least one question from each of the four lesson components. You may do additional questions for practice.

SC 2. Turn to "Chapter 7 Review" on pages 276 and 277. Complete questions 1, 2, 3, and 4. Then answer at least two questions from each of the four lesson components. You may do additional questions for practice.

Compare your answers in the Appendix.

Still not clear about some of the answers to the chapter review questions? Then contact your teacher to discuss any concepts that you may still be struggling with.

Are You Ready?

This unit covered many concepts and skills. You've done a lot of work to arrive so close to the end of the unit, wouldn't you agree? But are you ready to show your mastery of the concepts and skills of this unit? Are you really ready to take the challenge of a unit test?

You may feel you are ready but would like to complete a practice test before doing one for marks. Well, here's your chance. This unit was based on two chapters in your textbook. The following Self-Check questions consist of selected questions from the two textbook "Practice Tests."

Challenge yourself and complete the Self-Check questions as you would a real unit test. At the end, see how well you do when you compare your answers. Review any unit sections you still might have questions about at this point.

Self-Check

SC 3. Turn to pages 190 and 191 of your textbook, and complete just the odd-numbered questions of the "Practice Test."

SC 4. Turn to pages 278 and 279 of your textbook, and complete just the odd-numbered questions of "Practice Test."

Compare your answers in the Appendix.

Assignment

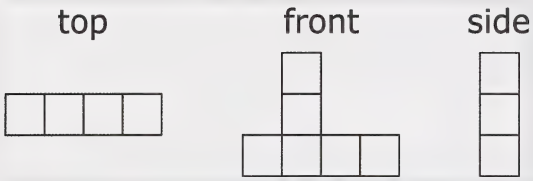
Check with your teacher about a unit test.

Congratulations! You've done it again: You have finished another unit of study. Way to Go! You've earned a hand of applause. (Just for fun, go to the Math 8 Multimedia DVD, and give yourself an "applause.")

Appendix

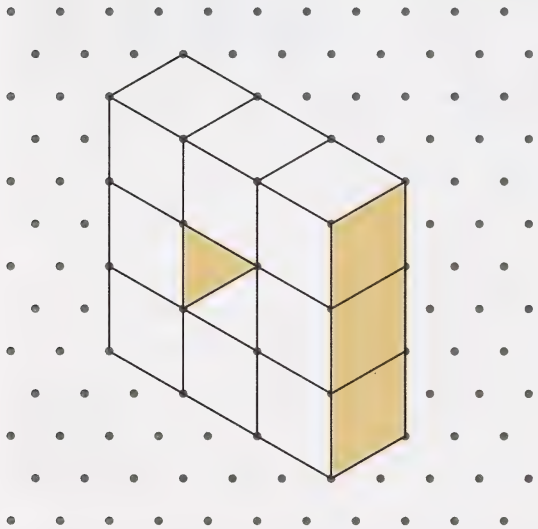
Lesson 1

SC 1.

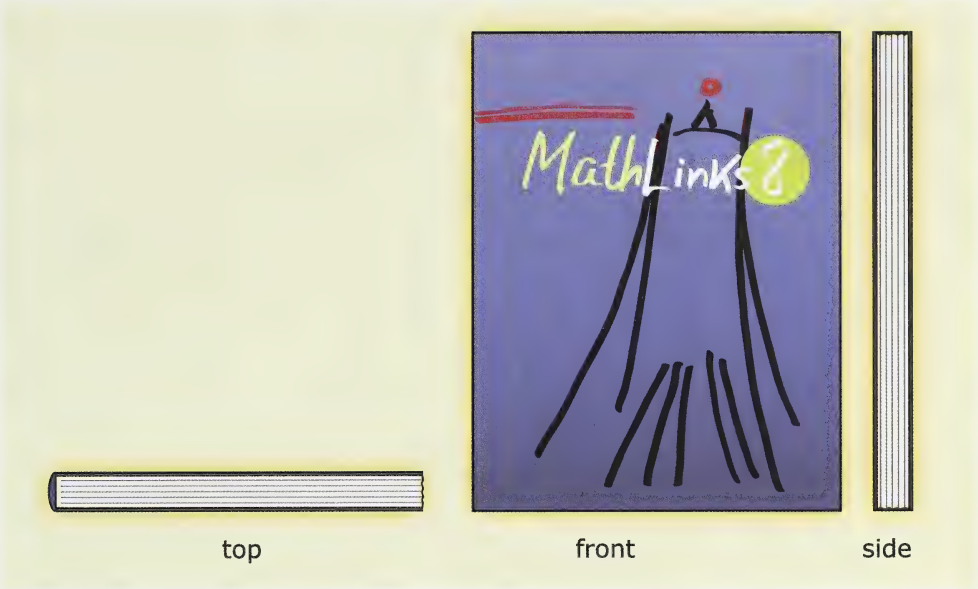


Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 220. Reproduced by permission.

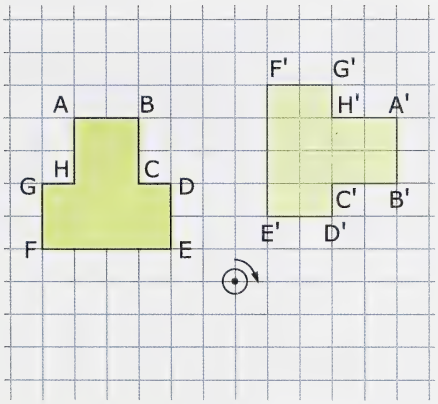
SC 2.



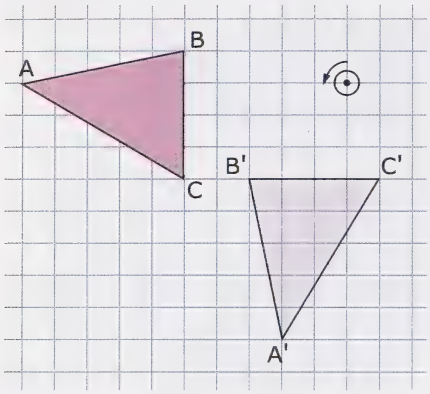
SC 3. The following are drawings of the top, front, and side views after rotation.



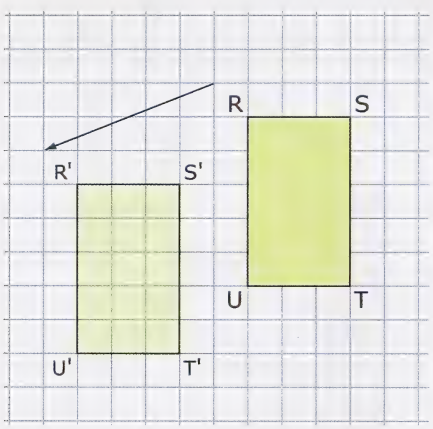
SC 4.a.



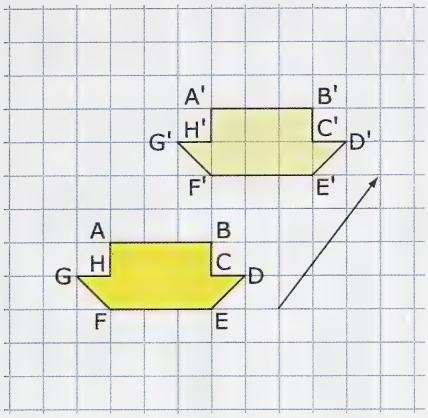
b.



SC 5.a. Move each vertex five units left and two units down.

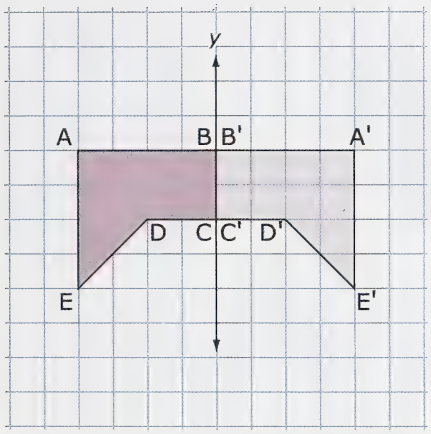


b. Move each vertex three units right and four units up.

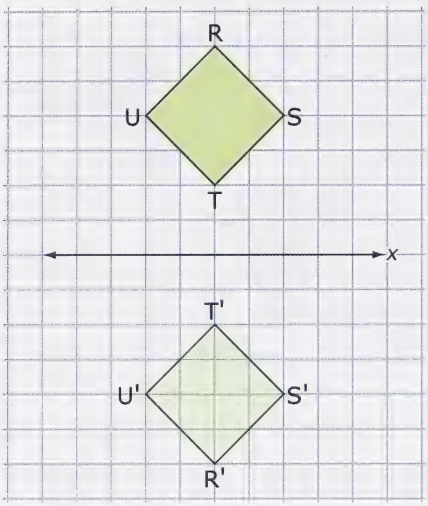


SC 6.

a.

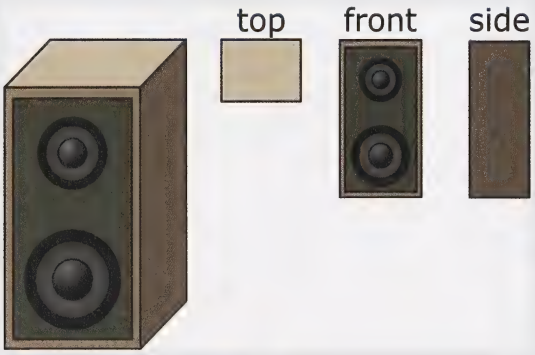


b.

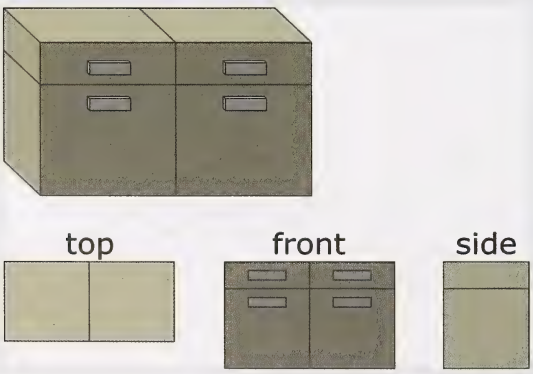


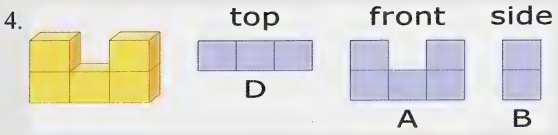
SC 7.

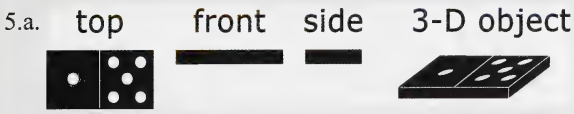
3.a.

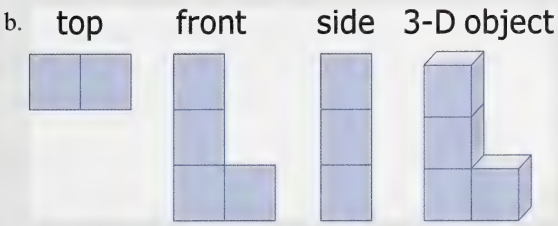


c.

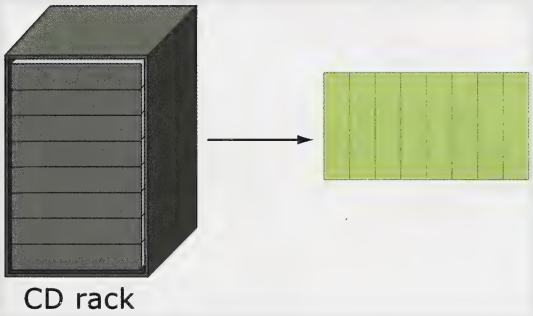








7. The object that will have a front view as shown after a rotation of 90° clockwise is b)—the CD rack.



MathLinks 8: Solutions CD (Toronto: McGraw-Hill Ryerson, 2008) 167-168. Reproduced by permission.

SC 8.

8. Answers will vary. Example: a chair and a bookcase.

Chair: top front side

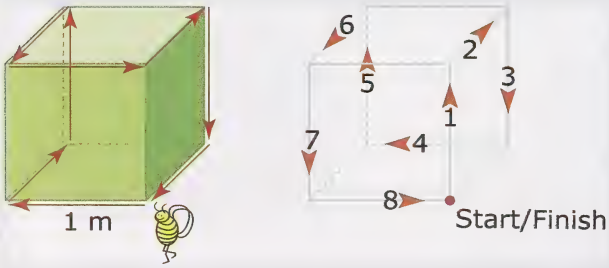
Bookcase:

top front side

9.b.

Going Beyond

11.a.There are two possible pathways.



b. The farthest distance that the bumblebee can travel is 8 m.

MathLinks 8: Solutions CD (Toronto: McGraw-Hill Ryerson, 2008) 169-170. Reproduced by permission.

Lesson 2

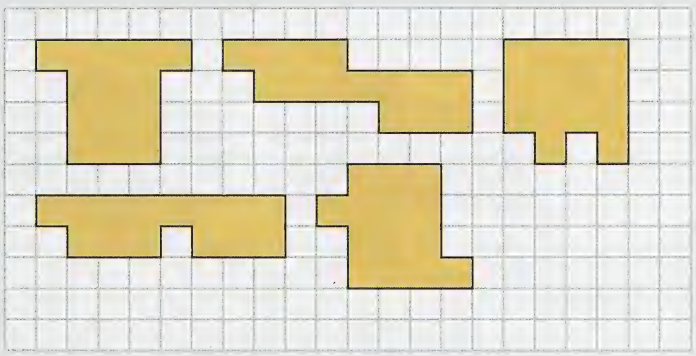
SC 1. A net requires that it has the same number of faces as the 3-D object it is supposed to represent. A cube has six faces, so Net A has the required number of faces to be a net of a cube. But more evidence is needed, since some nets with the required six faces will not work. The needed evidence comes by trying to use the net to form a cube.

When Net A is cut out and folded properly, it does form a cube. So, Net A does work as a net for a cube.

Using the logic that the number of faces of a 3-D object and its net has to be the same, it is clear that Net B will **not** work for a cube. It has only five faces, which is one less than needed to make a cube, which has six faces.

SC 2.

1. The nets will be as follows:



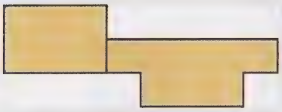
2. Your prediction is likely still based on limited experience, and your prediction may not turn out to be accurate. These can be tough to see, so if you want to find out for sure, draw the net on your grid paper, cut it out, and try to fold it into the shape you want.

3. No written answer is expected.

4. No, not all the nets create right rectangular prisms. The nets on the top right and bottom left do **not** create right rectangular prisms. Did you predict this result in textbook question 2?

5. How you roll the 3-D object and which object you used will affect the net you draw. Your net should have the same number of faces as your 3-D object.

Sample drawing:



[1]

[1] Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 226. Reproduced by permission.

SC 3. The following is a net for an unopened soup can.

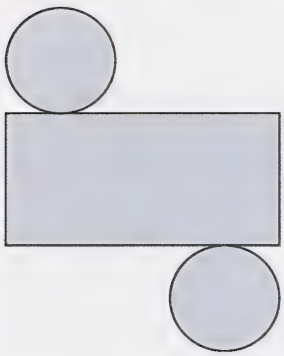
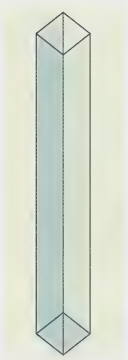


image: Bruce McAskill et al., *MathLinks 8 Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 226. Reproduced by permission.

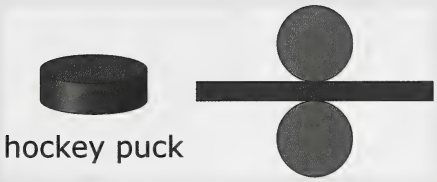
There should be two disks attached to a rectangle. The width of the rectangle should be equal to the height of your soup can. The length of the rectangle should be equal to the circumference of the disks.

SC 4. The object made from the net is a right rectangular prism having a square base.



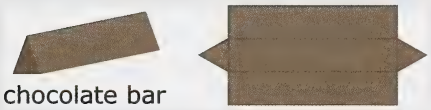
SC 5.

3.a.



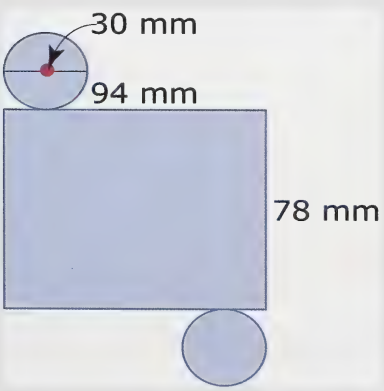
Note: The circles can be anywhere along the rectangle; they don't have to be in the middle.

b.

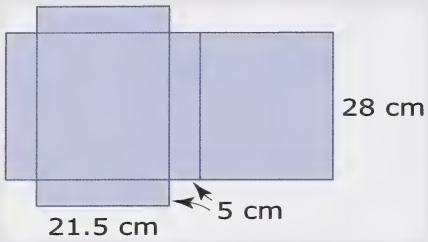


c.

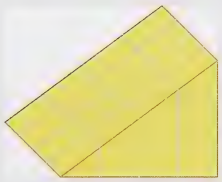
4.a. Answers may vary. Example:



b. Answers may vary. Example:

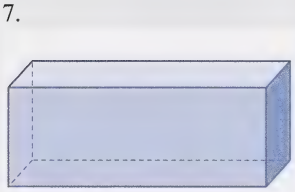


6.a.

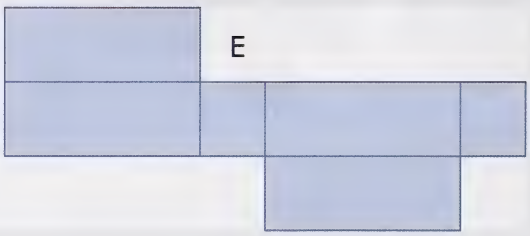


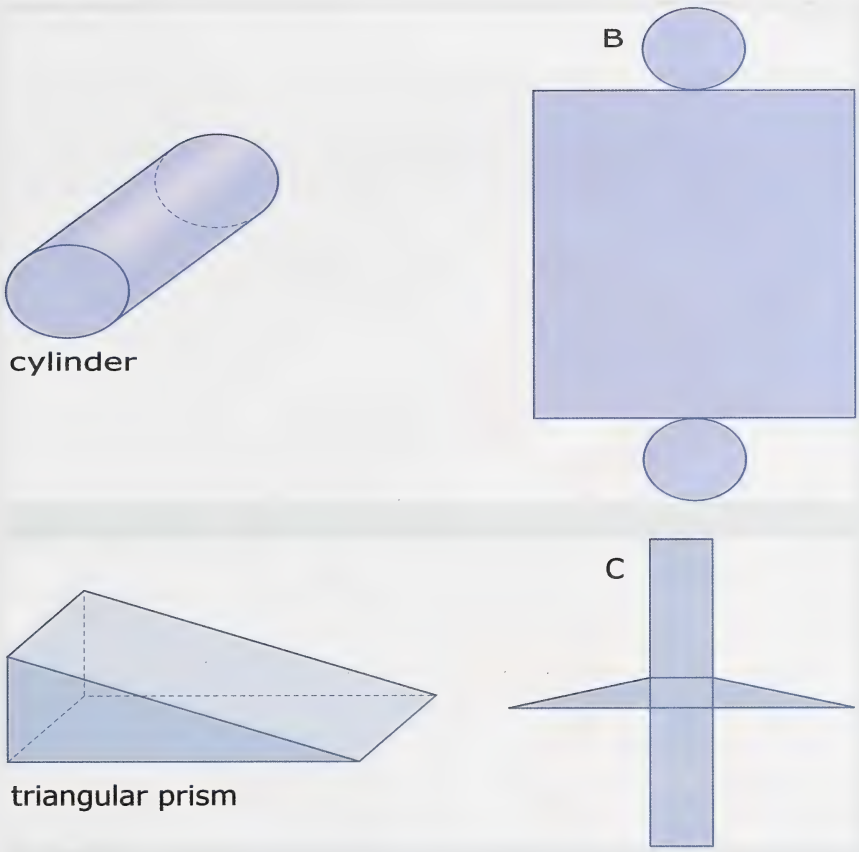
b. This object is a triangular prism.

7.



rectangular prism





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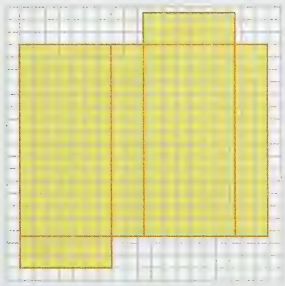
SC 6.

8. The net shown can be used to construct a box measuring 15.5 cm by 7 cm by 2.5 cm.



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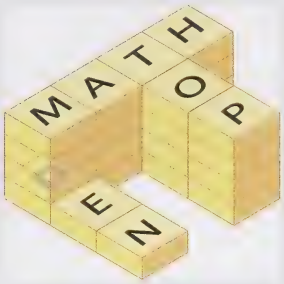
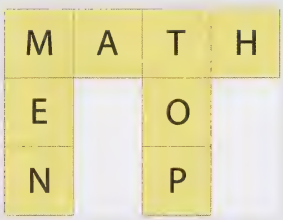
Going Beyond

11.a. The tiles should be no more than four tiles high anywhere but may be lower. The exposed letters should form words. The following answers are based on a sample 3-D object made from stacked tiles made playing the word game. The 3-D object is depicted by an isometric drawing.



b. Only the top tile in each pile is visible. The letters make words.



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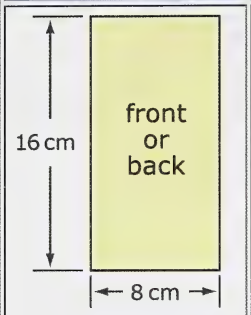
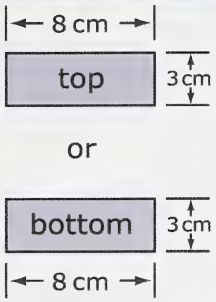
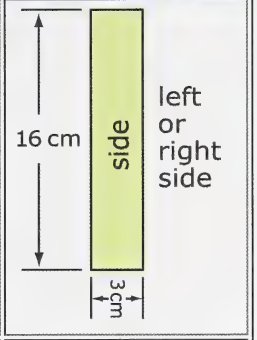
Lesson 3

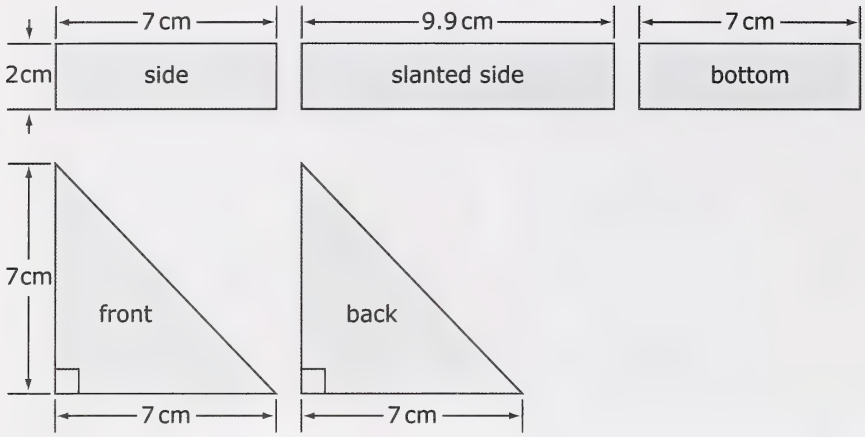
SC 1.

front or back	top or bottoms	left or right side
		
$A = l \times w$ $= 16 \times 8$ $= 128$ The area of the front or back is 128 cm^2.	$A = l \times w$ $= 8 \times 3$ $= 24$ The area of the top or bottom is 24 cm^2.	$A = l \times w$ $= 16 \times 3$ $= 48$ The area of the left or right sides is 48 cm^2.
The surface area is the sum of the areas of the faces of the prism. This sum is equal to the sum of the following: • area of front and back • area of top and bottom • area of left and right sides		
Area of front and back $A = 128 \times 2$ $= 256$	Area of top and bottom $A = 24 \times 2$ $= 48$	Area of left and right side $A = 48 \times 2$ $= 96$

surface area $= 256 + 48 + 56$
$= 400$

The surface area of the right rectangular prism is 400 cm^2.

SC 2. The following are faces of the prism.



The upright side and bottom are the same size and shape. The two triangles are the same size and shape.

Area of upright side:

$A = l \times w$
$= 2 \times 7$
$= 14$

Area of triangular side:

$A = (b \times h) \div 2$
$= (7 \times 7) \div 2$
$= 49 \div 2$
$= 24.5$

Area of slanted side:

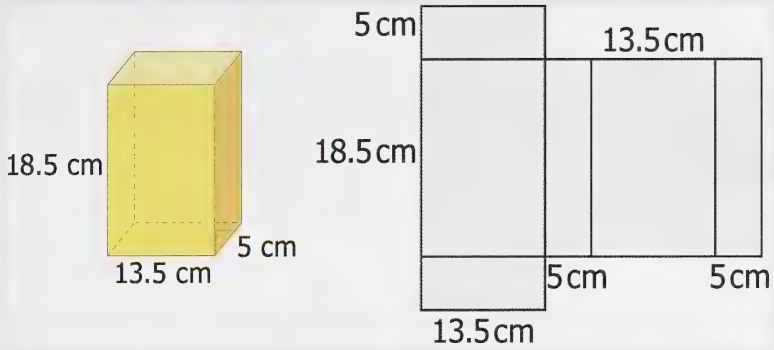
$A = l \times w$
$= 2 \times 9.9$
$= 19.8$

$$\begin{aligned}\text{surface area} &= (2 \times \text{area of upright side}) + (2 \times \text{area of triangle}) + (\text{area of slanted side}) \\ &= (2 \times 14) + (2 \times 24.5) + 19.8 \\ &= 28 + 49 + 19.8 \\ &= 96.8\end{aligned}$$

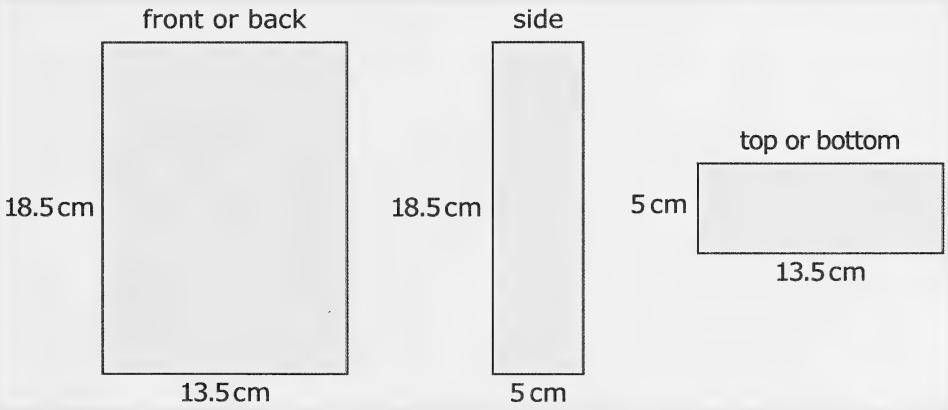
The surface area of the prism is 96.8 cm^2.

SC 3.

3. Draw the net for the rectangular prism.



Calculate the area (A) of each type of face.



$A = 13.5 \times 18.5$
$= 249.75$

$A = 18.5 \times 5$
$= 92.5$

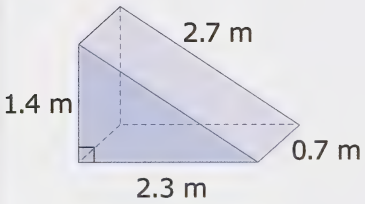
$A = 13.5 \times 5$
$= 67.5$

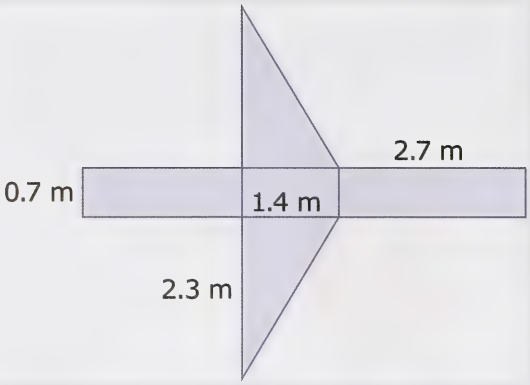
To determine the surface area, double each face value and add the answers.

$$SA = (2 \times 249.75) + (2 \times 92.5) + (2 \times 67.5)$$
$$= 499.5 + 185 + 135$$
$$= 819.5$$

The surface area is 819.5 cm^2.

5. Draw a net for the triangular prism.





Find the areas of each of the five faces. The two triangular faces will have the same area.

2 triangular faces	smallest rectangle	largest rectangle	remaining rectangle
$A = 2 \times \frac{2.3 \times 1.4}{2}$	$A = 0.7 \times 1.4$	$A = 0.7 \times 2.7$	$A = 0.7 \times 2.3$
$= 3.22$	$= 0.98$	$= 1.89$	$= 1.61$

Find the surface area by adding the areas of the five faces.

$$\begin{aligned}\text{surface area} &= 3.22 + 0.98 + 1.89 + 1.61 \\ &= 7.7\end{aligned}$$

The surface area is 7.7 m^2.

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SC 4.

7. The rectangular prism is made up of a pair of each of three different faces. Add the areas of the three faces, and then double the value to find the total surface area.

$$\begin{aligned}\text{surface area} &= 2 \times (20 + 12 + 15) \\ &= 2 \times 47 \\ &= 94\end{aligned}$$

The surface area of the rectangular prism is 94 mm^2.

8.a. The roof of the greenhouse is built from four glass faces. There are two congruent rectangles and two congruent triangles.

b. Calculate the surface area by doubling the areas of the rectangle and triangle and then finding the sum.

2 triangles

$$A = 2 \times \frac{1.8 \times 0.6}{2}$$
$$= 1.08$$

2 rectangles

$$A = 2 \times (2.4 \times 1.1)$$
$$= 5.28$$

Find the surface area by adding the areas of the four faces.

$$\text{surface area} = 1.08 + 5.28$$
$$= 6.36$$

Paco needs to buy 6.36 m^2 of glass.

10. Find the surface area of the ramp by adding the areas of the three rectangles and two triangles in the net.

2 triangles

$$A = 2 \times \frac{2 \times 0.9}{2}$$
$$= 1.8$$

smallest rectangle

$$A = 1.6 \times 0.9$$
$$= 1.44$$

medium-sized rectangle

$$A = 1.6 \times 2$$
$$= 3.2$$

largest rectangle

$$A = 1.6 \times 2.2$$
$$= 3.52$$

$$\textit{surface area} = 1.8 + 1.44 + 3.2 + 3.52$$
$$= 9.96$$

The ramp has a surface area of 9.96 m^2.

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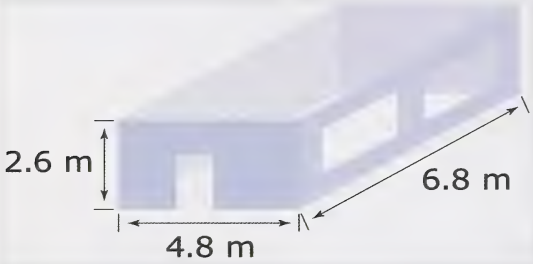
Going Beyond

16.a. Find the surface area of the walls and ceiling separately (since they require different priced paints).

Walls:



$$\text{surface area} = 2\times(2.6\times 4.8 + 2.6\times 6.8)$$
$$= 60.32$$

Type of Paint	Size of Paint Can	Cost
Wall paint	4L 1L	\$24.95 \$7.99
Ceiling paint	4L	\$32.95

Subtract 5 m^2 for doors and windows:

60.32 – 5 = 55.32

Ceiling:

4.8 × 6.8 = 32.64

Determine the number of litres (L) of paint for each region:

Walls: 55.32 ÷ 9.5 = 5.82 L
Ceiling: 32.64 ÷ 9.5 = 3.44 L

The walls require one 4-L can and two 1-L cans; the ceiling requires one 4-L can.

b) Answers will vary. Example:

Use the price chart to calculate the cost of the paint.

Wall paint:

$1 \times 24.95 + 2 \times 7.99 = \$40.93$

Ceiling paint:

$1 \times 32.95 = \$32.95$

The total cost before tax:

$40.93 + 32.95 = \$73.88$

At a tax rate of 12% (GST and PST), the total cost would be $1.12 \times 73.88 = \$82.75$.

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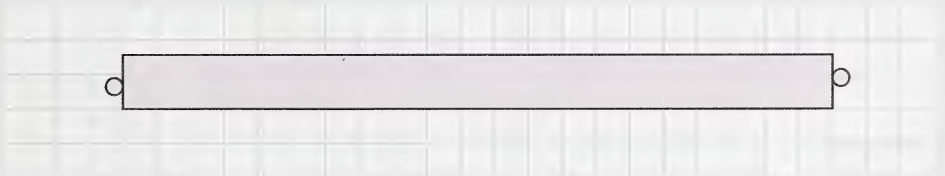
Lesson 4

SC 1.

1.a. Your drawings should look like this on 1-cm grid paper.



The circles have a diameter of 0.5 cm; and the rectangle is 21 cm long with a width of 1.6 cm.

b. In the net, the two circular faces are attached to the ends of the rectangle. The circular faces have a diameter of 0.5 cm. The rectangle is 21 cm long and about 1.6 cm wide.

2. You would use your knowledge about circles as follows:

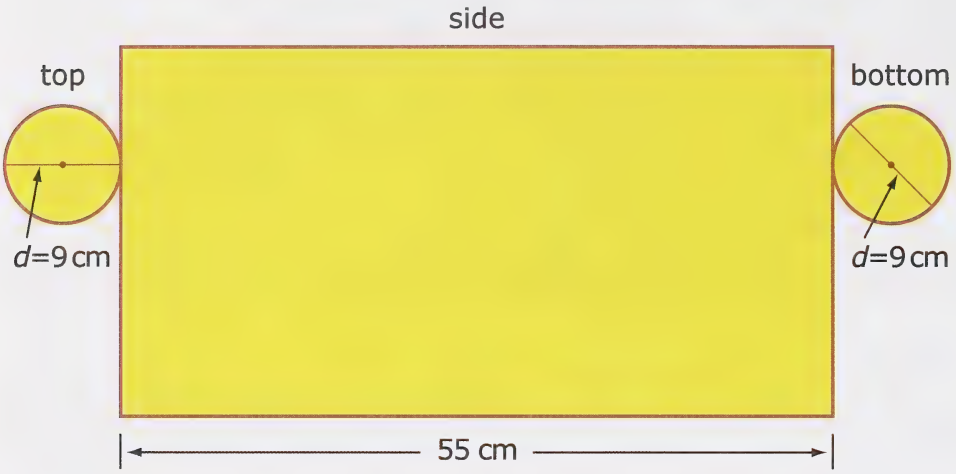
- Calculate the area of one circular base using the formula $A = \pi r^2$.
- Add the areas of the two circular bases.
- Use the formula for circumference of a circle, $C = \pi d$, to find the width of the rectangle.

3. For whatever method or strategy you used, you should end up with a surface area of 33.37 cm^2.

So, 33.37 cm^2 of plastic (sheet) is needed to make the glow stick.

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 239. Reproduced by permission.

SC 2.



Area of circular top and bottom

The diameter of the circular top is 9 cm. $9 \div 2 = 4.5$. Its radius is 4.5 cm.

$$A = \pi r^2$$
$$\approx 3.14 \times 4.52$$
$$\approx 63.585$$

The area of the top is approximately 63.585 cm^2.

$2 \times 64.585 = 127.17$

The area of both top and bottom is 127.17 cm^2.

Area of rectangle (based on circumference of circular top or bottom)

$$\begin{aligned} A &= l \times w \\ &= (\pi \times d) \times w \\ &\approx (3.14 \times 9) \times 55 \\ &\approx 1554.3 \end{aligned}$$

The area of the rectangle is approximately 1554.3 cm^2.

Surface area of cylinder

$$\begin{aligned} SA &\approx 127.17 + 1554.3 \\ &\approx 1681.5 \end{aligned}$$

The surface area of the cylinder is 1681.5 cm^2 to the nearest tenth of a square centimetre.

SC 3. The surface area of the cylindrical waste bucket is 1837 cm^2 (approximately).

Area of circular bottom (there is no top since there is no lid)

The diameter of the circular bottom is 18 cm.

$18 \div 2 = 9$

Its radius is 9 cm.

$$\begin{aligned} A &= \pi r^2 \\ &\approx 3.14 \times 92 \\ &\approx 254.469 \end{aligned}$$

The area of the bottom is approximately 254.5 cm^2.

Area of rectangle (based on circumference of circular top or bottom)

$$\begin{aligned} A &= l \times w \\ &= (\pi \times d) \times w \\ &\approx (3.14 \times 18) \times 28 \\ &\approx 1583.4 \end{aligned}$$

The area of the rectangle is approximately 1583.4 cm^2.

Surface area of cylinder

$SA \approx 254.5 + 1583.4$
≈ 1837.9

The surface area of the cylinder is 1838 cm^2 to the nearest square centimetre.

SC 4.

4. Estimate surface area by rounding decimal values to integer values before computing.

$SA = 2 \times (\pi \times r^2) + (\pi \times d \times h)$

a.

estimated SA value

$SA = 2 \times 3 \times 4^2 + 7 \times 3 \times 30$
$= 96 + 630$
$= 726$

calculated SA value

$SA = 2 \times 3.14 \times 3.5^2 + 7 \times 3.14 \times 30$
$\approx 76.93 + 659.4$
≈ 736.33

The surface area for the cylinder is 736.3 cm^2.

b.

estimated SA value

$SA = 2 \times 3 \times 10^2 + 20 \times 3 \times 20$
$= 600 + 1200$
$= 1800$

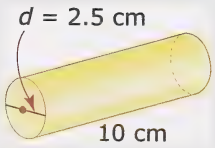
calculated SA value

$SA \approx 2 \times 3.14 \times 10^2 + 20 \times 3.14 \times 22$
$\approx 628 + 1381.6$
≈ 2009.6

The surface area for the cylinder is 2009.6 cm^2.

6.

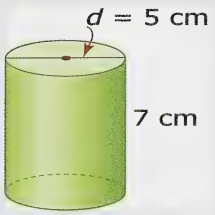
a.



$SA = 2 \times \left(\pi \times r^2\right) + \left(\pi \times d \times h\right)$
$\approx 2 \times 3.14 \times 1.25^2 + 3.14 \times 2.5 \times 10$
$\approx 9.8125 + 78.5$
≈ 88.3125

The surface area is 88.31 cm^2.

b.



$$SA = 2\times\left(\pi\times r^2\right)+\left(\pi\times d\times h\right)$$
$$\approx 2\times 3.14\times 2.5^2+3.14\times 5\times 7$$
$$\approx 39.25+109.9$$
$$\approx 149.15$$

The surface area is 149.15 cm^2.

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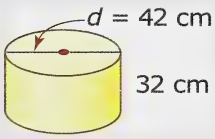
SC 5.

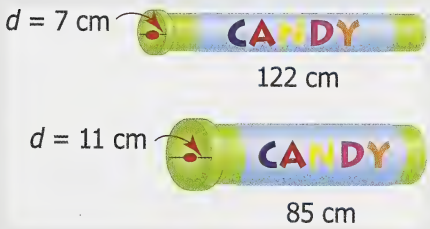
8. Find the surface area of the cylinder, excluding the bottom.



$$SA = \left(\pi\times r^2\right)+\left(\pi\times d\times h\right)$$
$$\approx 3.14\times 21^2+3.14\times 42\times 32$$
$$\approx 1384.74+4220.16$$
$$\approx 5604.9$$

Anu will require 5604.9 cm^2 of material to re-cover the stool.

9. Find the surface area of each container.



7-cm diameter container:

$$\begin{aligned} SA &= 2\times\left(\pi\times r^2\right)+\left(\pi\times d\times h\right) \\ &\approx 2\times\left(3.14\times 3.5^2\right)+\left(3.14\times 7\times 122\right) \\ &\approx 76.93+2681.56 \\ &\approx 2758.49 \end{aligned}$$

11-cm diameter container:

$$\begin{aligned} SA &= 2\times(\pi\times r^2)+(\pi\times d\times h) \\ &\approx 2\times\left(3.14\times 5.5^2\right)+\left(3.14\times 11\times 85\right) \\ &\approx 189.97+2935.9 \\ &\approx 3125.87 \end{aligned}$$

The container with a diameter of 11 cm required more plastic to make. It has a surface area of 3125.87 cm^2 in comparison to 2758.49 cm^2 for the container with a diameter of 7 cm.

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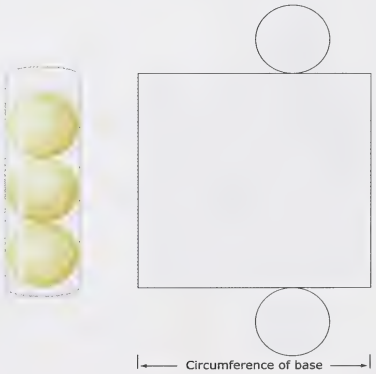
Going Beyond

11. In reality, tennis ball containers are sized to allow some extra room for the balls. However, the following solution is based on a tight fit for the tennis balls in the container. So, the diameter of the tennis ball container is considered to be equal to the diameter of a tennis ball, and the height of the container is equal to three times the diameter of a tennis ball.

Find the height of the cylinder needed to hold three balls with a diameter of 7 cm.

Height: 7 cm × 3 = 21 cm



Determine the surface area of a cylindrical container that has a height of 21 cm.

$$\begin{aligned} SA &= 2\times\left(\pi\times r^2\right)+\left(\pi\times d\times h\right) \\ &\approx 2\times 3.14\times 3.5^2+3.14\times 7\times 21 \\ &\approx 76.93+461.58 \\ &\approx 538.51 \end{aligned}$$

The surface area of the tennis ball container is 538.51 cm^2.

Lesson 5

SC 1.

The volume of a 3-D object is equal to the number of cubic centimetres you can fit inside the object.

SC 2.a. There are six squares making up the top base of the prism, so there are six blocks making up the top base.

b. You know that the opposite faces of a right rectangular prism are the same size (and shape). So from that relation, you'd know that the number of blocks making up the bottom face is equal to the number making up the top face. That would let you conclude that the bottom base is made of six cubes—just like the top base.

Here's another way to look at it. This is the view of the prism from below.

The bottom base of the prism is made up of six squares, so it is made up of six blocks.

SC 3. There are six cubes in the lowest layer of the prism. The prism has a height of 4 cm, so the prism has four horizontal layers of blocks each containing six cubes.

$6 \times 4 = 24$

There are 24 blocks in the prism. So the volume of the prism is 24 cm^3.

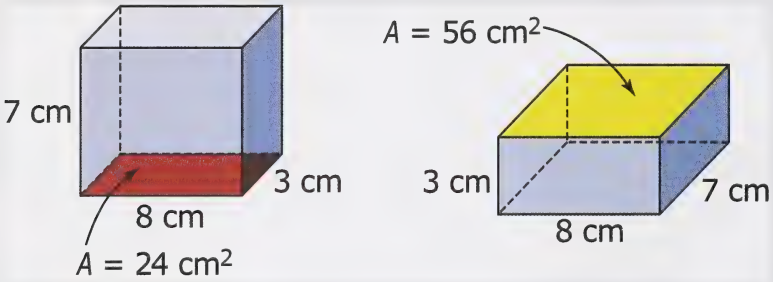
SC 4.

The volume of the cylinder is 880 cm^3.

$$\text{volume} = \text{area of base} \times \text{height}$$
$$V = 40 \times 22$$
$$= 880$$

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 340. Reproduced by permission.

SC 5. The boxes have equal volume. Your explanation may be based on orientation or calculations.



Think of the box on the left as a prism. Change its orientation by tipping it towards you until it comes to rest. The re-orientated prism is identical to the prism shape of the other box. Changing the orientation of a 3-D object never affects its volume.

You can support your conclusion with calculations instead.

Volume box on left side	Volume box on right side
Length of base is 8 cm and its width is 3 cm. Area of base is 8 × 3. Area of the base is 24 cm^2. volume = area of base x height = 24 × 7 = 168 Volume of box is 168 cm^3.	Length of base is 8 cm and its width is 7 cm. Area of base is 8 × 7. Area of the base is 56 cm^2. volume = area of base × height = 56 × 3 = 168 Volume of box is 168 cm^3.

The calculations show the volumes are equal.

Adapted from Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 389. Reproduced by permission.

SC 6.

3.a. The prism is a right rectangular prism. The area of the rectangular base is 15 cm^2. The height of the prism is 4 cm.

$$\begin{aligned}\text{volume} &= \text{area of base} \times \text{height of prism} \\ V &= 15 \times 4 \\ &= 60\end{aligned}$$

The volume of the right rectangular prism is 60 cm^3.

c. The prism is a right triangular prism. The area of the triangular base is 96 cm^2.

The height of the prism is 20 cm.

$$\begin{aligned}\text{volume} &= \text{area of base} \times \text{height of prism} \\ V &= 96 \times 20 \\ &= 1920\end{aligned}$$

The volume of the right triangular prism is 1920 cm^3.

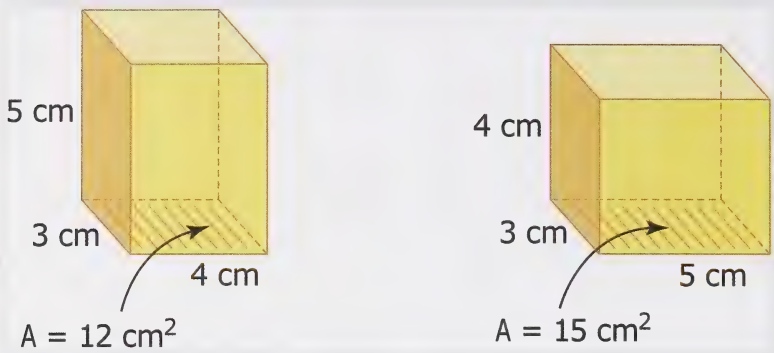
4.a. The volume of the right prism is 96 cm^3.

$$\begin{aligned}\text{volume} &= \text{area of base} \times \text{height of prism} \\ V &= 12 \times 8 \\ &= 96\end{aligned}$$

c. The volume of the right prism is 126 cm^3.

$$\begin{aligned}\text{volume} &= \text{area of base} \times \text{height of prism} \\ V &= 14 \times 9 \\ &= 126\end{aligned}$$

5.a.



The prism on the left is a right rectangular prism. The area of the rectangular base is 12 cm^2. The height of the prism is 5 cm.

$$\begin{aligned}\text{volume} &= \text{area of base} \times \text{height of prism} \\ V &= 12 \times 5 \\ &= 60\end{aligned}$$

The volume of the right rectangular prism is 60 cm^3.

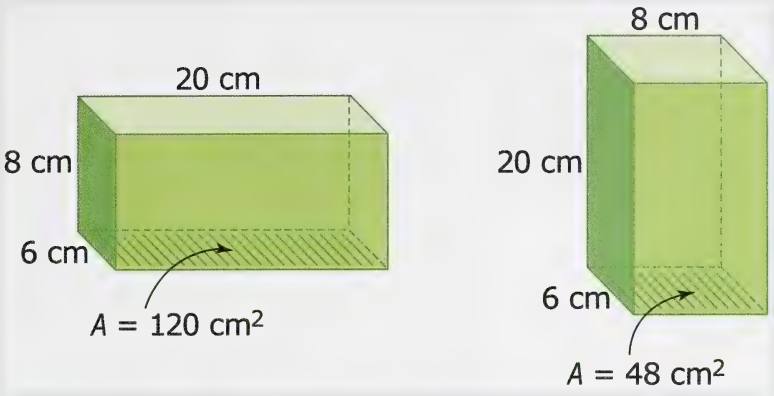
The prism on the right is a right rectangular prism. The area of the rectangular base is 15 cm^2. The height of the prism is 4 cm.

$$\begin{aligned}\text{volume} &= \text{area of base} \times \text{height of prism} \\ V &= 15 \times 4 \\ &= 60\end{aligned}$$

The volume of the right rectangular prism is 60 cm^3.

The two prisms have the same volume. You could expect this. The prisms only differ in orientation.

b.



The first prism is a right rectangular prism. The area of the rectangular base is 120 cm^2. The height of the prism is 8 cm.

$$\begin{aligned}\text{volume} &= \text{area of base} \times \text{height of prism} \\ V &= 120 \times 8 \\ &= 960\end{aligned}$$

The volume of the right rectangular prism is 960 cm^3.

The second prism is a right rectangular prism. The area of the rectangular base is 48 cm^2. The height of the prism is 20 cm.

$$\begin{aligned}\text{volume} &= \text{area of base} \times \text{height of prism} \\ V &= 48 \times 20 \\ &= 960\end{aligned}$$

The volume of the right rectangular prism is 960 cm^3.

The two prisms have the same volume. They differ only in orientation so the volumes should be the same.

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SC 7.

7.a. To determine the height of a right rectangular prism, given the volume and the area of the base, divide the volume by the area of the base.

$$\text{height of prism} = \text{volume} \div \text{area of base}$$
$$h = 32 \div 8$$
$$h = 4$$

The height of the prism is 4 cm.

8. To find the volume of a rectangular prism, multiply the area of the base by the number of layers. The base consists of 15 centimetre cubes, so the area of the base is 15 cm^2.

$$\text{volume} = \text{area of base} \times \text{number of layers}$$
$$V = 15 \times 5$$
$$= 75$$

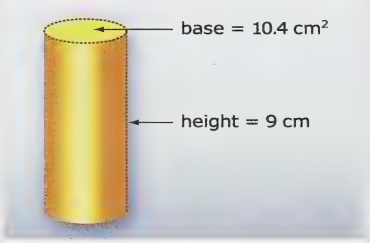
The volume of the prism is 75 cm^3.

10. To find out how much water can be put in the trough before it overflows, find the volume of the triangular prism. The area of the triangular base is 1250 cm^2. The height of the prism is 100 cm.

$$\text{volume} = \text{area of base} \times \text{height of prism}$$
$$V = 1250 \times 100$$
$$= 125\ 000$$

The trough can hold 125 000 cm^3 of water before it overflows.

11. The shape of the can is a cylinder. To find the volume of soup in the can, multiply the area of the circular base (10.4 cm^2) by the height of the soup in the can (9 cm).



volume = area of base × height of soup in the cylinder

$$V = 10.4 \times 9$$
$$= 93.6$$

The volume of soup in the can is 93.6 cm^3.

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Going Beyond

16.a. The number of cubes in each structure:

- Structure 1: 10
- Structure 2: 14
- Structure 3: 15

b. To determine the minimum number of cubes needed to complete each structure, find the sum of the number of cubes needed to complete each layer. The minimum number of cubes needed to complete the whole structures:

- Structure 1: 17
- Structure 2: 22
- Structure 3: 30

c. To determine the number of cubes in each completed structure, multiply the number of cubes in the completed base by 3—the number of layers in the completed structures. The total number of cubes in each completed structure:

- Structure 1: 27
- Structure 2: 36
- Structure 3: 45

Note that for Structure 1, the value in part a plus the value in part b equals the value in part c. This relation also applies to Structures 2 and 3.

d. The volume of each prism:

- Structure 1: 216 cm^3
- Structure 2: 288 cm^3
- Structure 3: 360 cm

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Lesson 6

SC 1.

1. Volume for a prism equals the product of the area of the base and the prism's height.

volume = area of base × height

2. Area of base equals the area of a rectangle having a length of 9 cm and a width of 5 cm.

$$A = 9 \times 5$$
$$= 45$$

The area of the base is 45 cm^2.

The volume of the prism equals the area of the base multiplied by the prism's height.

$$\text{volume} = \text{area of base} \times \text{prism height}$$
$$= 45 \times 4$$
$$= 180$$

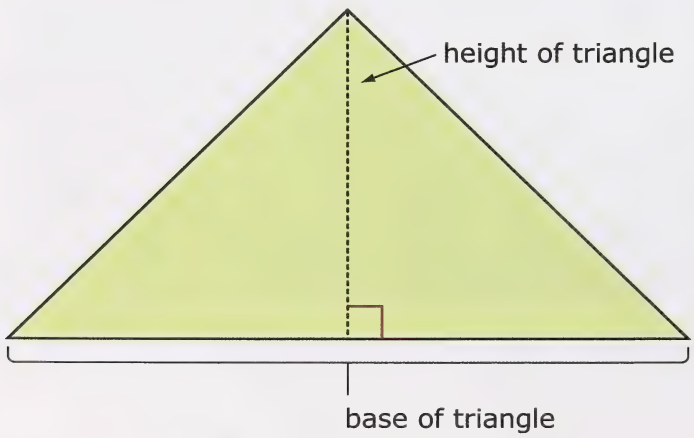
The volume of the prism is 180 cm^3.

3. Area of prism base equals the area of a triangle having a base length of 8 cm and a height of 5 cm.

To find the area of the base, you need to remember the formula for the area of a triangle.

area = base of triangle × height of triangle ÷ 2

Remember that the base and the height of the triangle are perpendicular.



$$\begin{aligned} A &= b \times h \div 2 \\ &= 8 \times 5 \div 2 \\ &= 40 \div 2 \\ &= 20 \end{aligned}$$

The area of the base is 20 cm^2.

The volume of the prism equals the area of the base multiplied by the prism's height.

$$\begin{aligned} \text{volume} &= \text{area of base} \times \text{prism height} \\ &= 20 \times 4 \\ &= 80 \end{aligned}$$

The volume of the triangular prism is 80 cm^3.

SC 2.

a. The volume of the right rectangular prism is 170 cm^3.

$$\begin{aligned} V &= l \times w \times h \\ &= 8.5 \times 4 \times 5 \\ &= 170 \end{aligned}$$

b. The volume of the right rectangular prism is 27 m^3.

$$\begin{aligned} V &= s \times s \times s \\ &= 3 \times 3 \times 3 \\ &= 27 \end{aligned}$$

SC 3.

The base of the prism is a special triangle called a right triangle. So, the side length of 5 mm can be thought of as the height of the base as a triangle.

$$\begin{aligned} V &= (\text{base} \times \text{height} \div 2) \times \text{height of prism} \\ &= (9 \times 5 \div 2) \times 8 \\ &= (45 \div 2) \times 8 \\ &= 22.5 \times 8 \\ &= 180 \end{aligned}$$

The volume of the right triangular prism is 180 mm^3.

Suppose you had used 9 mm as the height of the base; then your solution would have started this way.

$$V = (\text{base} \times \text{height} \div 2) \times \text{height of prism}$$
$$= (5 \times 9 \div 2) \times 8$$

You can see that the effect of using a different side for the height merely interchanges the numbers 5 and 9. You'd still get for the same value for the volume.

SC 4.

The total volume of the box is 19 200 cm^3.

$$V = l \times w \times h$$
$$= 40 \times 24 \times 20$$
$$= 960 \times 20$$
$$= 19\ 200$$

Each child will receive a volume of 4800 cm^3 of blocks.

$19\ 200 \div 4 = 4800$

If the blocks were, for example, 4-cm cubes, then the volume of each building block would have been 16 cm^3.

$4800 \div 16 = 75$

Each child would receive 75 building blocks.

SC 5.

4.a. The volume of the right rectangular prism is 15 m^3.

volume of a right rectangular prism = area of rectangular base × height of prism
volume of a right rectangular prism = (length × width) × height

$$V = l \times w \times h$$
$$V = 1 \times 3 \times 5$$
$$V = 15$$

b. The volume of the right rectangular prism is 792 m^3.

volume of a right rectangular prism = area of rectangular base × height of prism
volume of a right rectangular prism = (length × width) × height

$V = l \times w \times h$
$V = 6 \times 11 \times 12$
$V = 792$

c. The volume of the right rectangular prism is 49.6 m^3.

volume of a right rectangular prism = area of rectangular base × height of prism
volume of a right rectangular prism = (length × width) × height

$V = l \times w \times h$
$V = 2 \times 4 \times 6.2$
$V = 49.6$

5.a. The volume of the right rectangular prism is 40 m^3.

volume of a right rectangular prism = area of rectangular base × height of prism
volume of a right rectangular prism = (length × width) × height

$V = l \times w \times h$
$V = 2 \times 2 \times 10$
$V = 40$

c. The volume of the right rectangular prism is 213.759 mm^3.

volume of a right rectangular prism = area of rectangular base × height of prism
volume of a right rectangular prism = (length × width) × height

$V = l \times w \times h$
$V = 11.7 \times 6.3 \times 2.9$
$V = 213.759$

6.c. The volume of the cube is 15.625 cm^3.

volume of a cube = area of square base × height
volume of a cube = (length × width) × height

$V = s \times s \times s$
$V = 2.5 \times 2.5 \times 2.5$
$V = 15.625$

7.a. The volume of the triangular prism is 294 cm^3.

volume of a triangular prism = area of triangular base × height of prism
volume of a triangular prism = (base × height ÷ 2) × height of prism

$V = (7 \times 7 \div 2) \times 12$
$V = 294$

b. The volume of the triangular prism is 133.65 m^3.

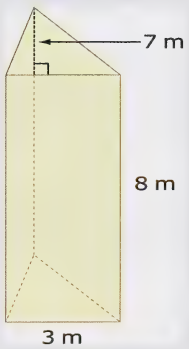
volume of a triangular prism = area of triangular base × height of prism
volume of a triangular prism = (base × height ÷ 2) × height of prism

$V = (8.1 \times 2.2 \div 2) \times 15$
$V = 133.65$

8.a. The volume of the triangular prism is 84 m^3.

volume of a triangular prism = area of triangular base × height of prism
volume of a triangular prism = (base × height ÷ 2) × height of prism

$V = (3 \times 7 \div 2) \times 8$
$V = 84$



b. The volume of the triangular prism is 1200 cm^3.

volume of a triangular prism = area of triangular base × height of prism
volume of a triangular prism = (base × height ÷ 2) × height of prism

$$V = (15 \times 8 \div 2) \times 20$$
$$V = 1200$$

9.a. volume of a right rectangular prism = area of rectangular base × height of prism

$$V = l \times w \times h$$
$$V = 10 \times 5 \times 12$$
$$V = 600$$

The volume of the right rectangular prism is 600 cm^3.
The prism is $\frac{1}{3}$ full.

$$\text{volume of contents} = \frac{1}{3} \times 600$$
$$= \frac{600}{3}$$
$$= 200$$

The volume of the contents is 200 cm^3.

b. volume of a right rectangular prism = area of rectangular base × height of prism

$$V = l \times w \times h$$
$$V = 15 \times 19 \times 3$$
$$V = 855$$

The volume of the right rectangular prism is 855 cm^3.
The prism is $\frac{3}{8}$ full.

$$\text{volume of contents} = \frac{3}{8} \times 855$$
$$= \frac{2565}{8}$$
$$= 320.625$$

The volume of the contents is 320.625 cm^3.

c. volume of a triangular prism = area of triangular base × height of prism

$V = (\text{base} \times \text{height} \div 2) \times \text{height of prism}$
$V = (260 \times 140 \div 2) \times 400$
$V = 7\ 280\ 000$

The volume of the right rectangular prism is 7 280 000 mm^3.
The prism is $\frac{3}{4}$ full.

$$\text{volume of contents} = \frac{3}{4} \times 7\ 280\ 000$$
$$= \frac{21\ 840\ 000}{4}$$
$$= 5\ 460\ 000$$

The volume of the contents is 5 460 000 mm^3.

10.a. volume of a cube = (length × width) × height

$V = s \times s \times s$
$V = 10 \times 10 \times 10$
$V = 1000$

The volume of the cube is 1000 cm^3.

The package is $\frac{4}{5}$ full. Therefore, $\frac{1}{5}$ of the package is empty.

$$\text{amount of empty space} = \frac{1}{5} \times 1000$$
$$= \frac{1000}{5}$$
$$= 200$$

There is 200 cm^3 of empty space in the box.

c. volume of a right triangular prism = (base × height ÷ 2) × height of prism

$$V = (12 \times 15 \div 2) \times 24$$
$$V = 2160$$

The volume of the right triangular prism is 2160 cm^3.

The object is $\frac{1}{6}$ full. Therefore, $\frac{5}{6}$ of the object is empty.

$$\text{amount of empty space} = \frac{5}{6} \times 2160$$
$$= \frac{10\,800}{56}$$
$$= 1800$$

There is 1800 cm^3 of empty space in the object.

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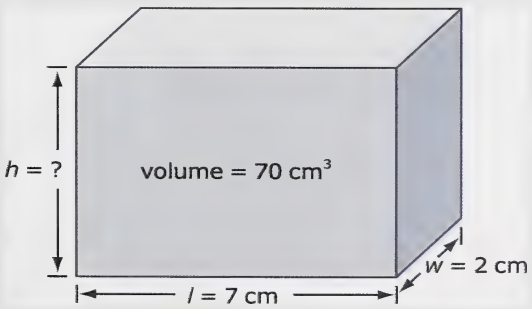
SC 6.

11.a. To determine the missing height of a right rectangular prism, divide the volume by the product of the length and width.

$$\text{height of prism} = \text{volume} \div (\text{length} \times \text{width})$$
$$h = 70 \div (7 \times 2)$$
$$h = 5$$

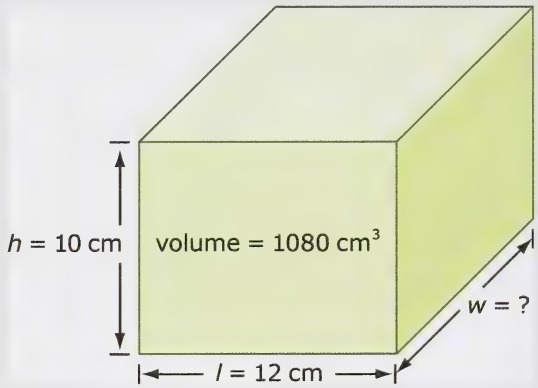
The missing height is 5 cm.



b. To determine the missing width of a right rectangular prism, divide the volume by the product of the length and height.

$$\text{width of prism} = \text{volume} \div (\text{length} \times \text{height})$$
$$w = 1080 \div (12 \times 10)$$
$$w = 9$$

The missing width is 9 cm.



c. To determine the missing length of a right rectangular prism, divide the volume by the product of the width and height.

$$\text{length of prism} = \text{volume} \div (\text{width} \times \text{height})$$
$$l = 1200 \div (15 \times 5)$$
$$l = 16$$

The missing length is 16 cm.

Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
7	2	5	70
12	9	10	1080
16	15	5	1200

12.a. To determine the missing height of a right triangular prism, multiply the volume by 2 and divide by the product of the base and the height of the base.

$$\text{height} = 2 \times \text{volume} \div (\text{base} \times \text{height of base})$$
$$h = 2 \times 70 \div (7 \times 2)$$
$$h = 10$$

The missing height is 10 cm.

b. To determine the missing height of the base of a right triangular prism, multiply the volume by 2 and divide by the product of the base and the height of the prism.

$$\text{height of base} = 2 \times \text{volume} \div (\text{base} \times \text{height of prism})$$
$$h = 2 \times 1080 \div (18 \times 10)$$
$$h = 12$$
$$\text{base} = 2 \times \text{volume} \div (\text{height of base} \times \text{height of prism})$$
$$b = 2 \times 700 \div (14 \times 5)$$
$$b = 20$$

The missing height is 12 cm.

c. To determine the missing base of a right triangular prism, multiply the volume by 2 and divide by the product of the height of the base and the height of the prism.

The missing base is 20 cm.

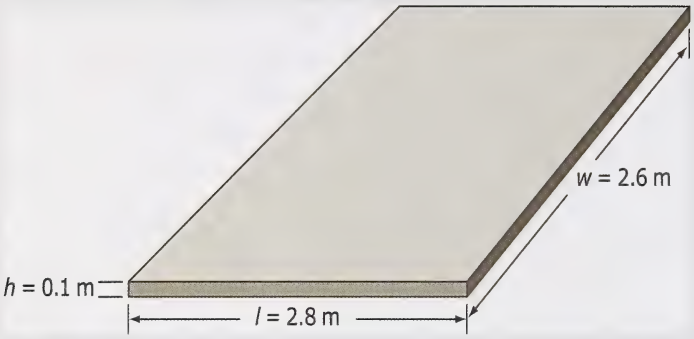
Base (cm)	Height of Triangle (cm)	Height of Prism (cm)	Volume (cm^3)
7	2	10	70
18	12	10	1080
20	14	5	700

13. To determine if the landscaper has enough gravel, find the volume of the patio. The patio's shape is a right rectangular prism. Convert the 10-cm measurement to metres: 10 cm = 0.1 m.

$$V = l \times w \times h$$
$$V = 2.8 \times 2.6 \times 0.1$$
$$V = 0.728$$

The volume of the patio is 0.728 m^3. She does not have enough gravel.

She needs $0.728 - 0.5 = 0.228$ m^3 more gravel.



$V = l \times w \times h$
$V = 120 \times 1.5 \times 0.1$
$V = 18$

14. Determine the volume of the vase.

volume of a right triangular prism = (base × height ÷ 2) × height of prism

$V = (15 \times 8 \div 2) \times 80$
$V = 4800$

The volume of the vase is 4800 cm^3.

15. The sidewalk is shaped like a right rectangular prism.

The volume of concrete used to make the sidewalk is 18 m^3.

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Going Beyond

24.
The following are sample solutions.

a. To determine the missing height of a right rectangular prism, divide the volume by the product of the length and width.

$$\text{height of prism} = \text{volume} \div (\text{length} \times \text{width})$$
$$h = 960 \div (40 \times 30)$$
$$h = 0.8$$

The depth of the water is 0.8 m.

b. Since the water drains at the rate of 60 m^3/h, after 2.5 hours, 60 × 2.5 → 150 m^3 of water would be drained. There would be 960 – 150 → 810 m^3 of water remaining.

To determine the missing height of a right rectangular prism, divide the volume by the product of the length and width:

$$\text{height of prism} = \text{volume} \div (\text{length} \times \text{width})$$
$$h = 810 \div (40 \times 30)$$
$$h = 0.675$$

The new depth of the water is 0.675 m.

c. Find the volume of water left using a height of 0.2 m.

$V = l \times w \times h$
$V = 40 \times 30 \times 0.2$
$V = 240$

The volume of water with a depth of 0.2 m is 240 m^3. This means that (810 – 240) → 570 m^3 of water has drained. To determine how long this took to drain, divide 570 by 60:

$570 \div 60 = 9.5$

The tank was draining for 9.5 + 2.5 = 12 h.

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Lesson 7

SC 1.

a. An estimated value of the volume of the cylinder is 120 cm^3.

$V = (\pi \times r^2) \times h$, where $h \approx 10$, $r \approx 2$, $\pi \approx 3$

Remember, since you are estimating, you may use rounded values.

$$\left(3 \times 2^2\right) \times 10$$
$$\approx (3 \times 4) \times 10$$
$$\approx 12 \times 10$$
$$\approx 120$$

Note that the units are cm^3 as you learned before, since you are multiplying the radius times the radius times the height (three different lengths) that gives cubed units.

b. The volume of the cylinder is 131.9 cm^3.

$$V = (\pi \times r^2) \times h$$
$$\approx (3.14 \times 2^2) \times 10.5$$
$$\approx 131.88$$

SC 2. The volume of the bin is 143 066 cm3, to the nearest cubic centimetre.

$$r = \mathrm{d} / 2$$
$$= 45 / 2$$
$$= 22.5$$

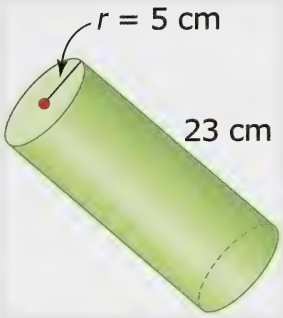
Remember that half the diameter is the radius of a circle. That's why the diameter is divided by 2.

$$V = (\pi \times r^2) \times h$$
$$\approx (3.14 \times 22.52) \times 90$$
$$\approx 143\,066.25$$

Note the size of the bin is hard to see from the diagram. But if you were standing beside it, the bin would likely come up to your waist.

SC 3.

4.a.

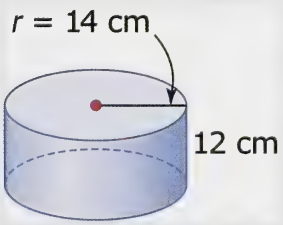


$$V = \left(\pi \times r^2\right) \times h$$
$$\approx \left(3.14 \times 5^2\right) \times 23$$
$$\approx 1805.5$$

volume of a cylinder = area of circular base × height of cylinder

The volume of the cylinder is 1805.5 cm^3.

b.

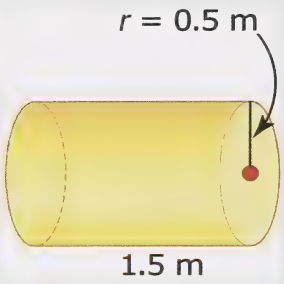


volume of a cylinder = area of circular base × height of cylinder

$$V = \left(\pi \times r^2\right) \times h$$
$$\approx \left(3.14 \times 14^2\right) \times 12$$
$$\approx 7385.28$$

The volume of the cylinder is 7385.28 cm^3.

c.



volume of a cylinder = area of circular base × height of cylinder

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 0.5^2\right) \times 1.5 \\ &\approx 1.1775 \end{aligned}$$

The volume of the cylinder is 1.1775 m^3.

5.a. volume of a cylinder = area of circular base × height of cylinder

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 5^2\right) \times 8 \\ &\approx 628 \end{aligned}$$

The volume of the cylinder is 628 cm^3.

c. volume of a cylinder = area of circular base × height of cylinder

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 1.1^2\right) \times 2.6 \\ &\approx 9.878\ 44 \end{aligned}$$

The volume of the cylinder is 9.878 44 m^3.

6.a. The diameter is 10 cm. The radius is half the diameter: $r = 10 \div 2 = 5$.

volume of a cylinder = area of circular base × height of cylinder

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 5^2\right) \times 20 \\ &\approx 1570 \end{aligned}$$

The volume of the cylinder is 1570 cm^3.

b. The diameter is 1 m. The radius is half the diameter: $r = 1 \div 2 = 0.5$.

volume of a cylinder = area of circular base × height of cylinder

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 0.5^2\right) \times 1 \\ &\approx 0.785 \end{aligned}$$

The volume of the cylinder is 0.785 m^3.

7.a. The diameter is 8 cm. The radius is half the diameter: $r = 8 \div 2 = 4$.

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 4^2\right) \times 12 \\ &\approx 602.88 \end{aligned}$$

The volume of the cylinder is 602.88 cm^3.

c. The diameter is 12 cm. The radius is half the diameter: $r = 12 \div 2 = 6$.

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 6^2\right) \times 37.5 \\ &\approx 4239 \end{aligned}$$

The volume of the cylinder is 4239 cm^3.

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SC 4.

8. To determine the height of the cylinder given the volume and area of the base, divide the volume by the area of the base.

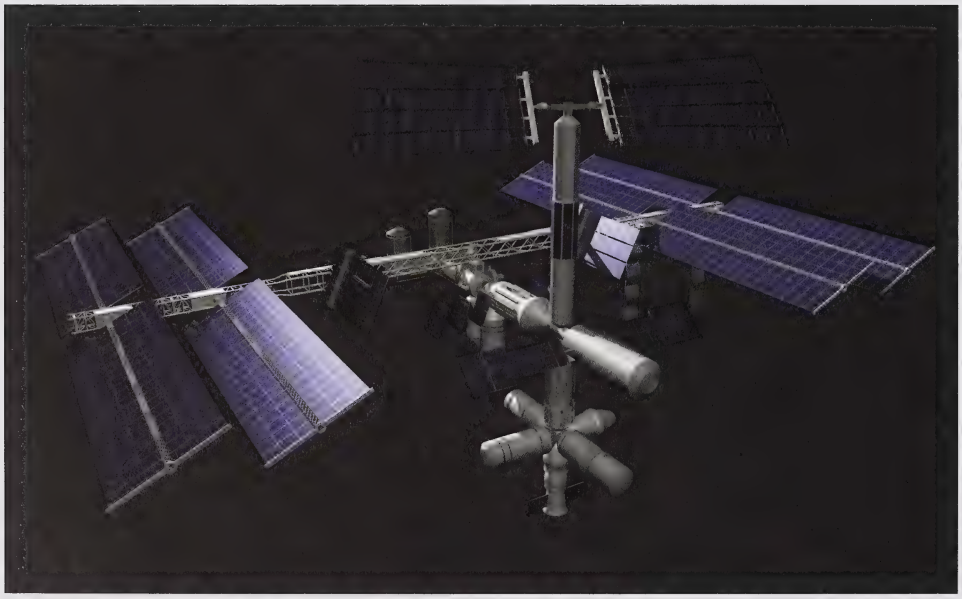
height of cylinder = volume of cylinder ÷ area of circular base

h = volume ÷ area of base
$h = 150 \div 48$
$h = 3.125$

The height of the cylinder is 3 cm, rounded to the nearest centimetre.

10. The radius is half the diameter.

volume of a cylinder = area of circular base × height of cylinder



© Michael Ransburg/shutterstock
The International Space Station has orbited Earth since the year 2000.

a. Zarya FGB element: $r = 4.1 \div 2 = 2.05$

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 2.05^2\right) \times 12.6 \\ &\approx 166.267\ 71 \end{aligned}$$

The volume of the cylindrical element is 166.267 71 m^3.

Unity Node 1 element: $r = 4.6 \div 2 = 2.3$

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 2.3^2\right) \times 5.5 \\ &\approx 91.3583 \end{aligned}$$

The volume of the cylindrical element is 91.3583 m^3.

Zvezda service module: $r = 4.2 \div 2 = 2.1$

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 2.1^2\right) \times 13.1 \\ &\approx 181.400\ 94 \end{aligned}$$

The volume of the cylindrical element is 181.400 94 m^3.

Z1 Truss element: $r = 4.2 \div 2 = 2.1$

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 2.1^2\right) \times 4.9 \\ &\approx 67.852\ 26 \end{aligned}$$

The volume of the cylindrical element is 67.852 26 m^3.

P6 Truss solar array element: $r = 10.7 \div 2 = 5.35$

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 5.35^2\right) \times 73.2 \\ &\approx 6578.824\ 38 \end{aligned}$$

The volume of the cylindrical element is 6578.824 38 m^3.

Destiny element: $r = 4.3 \div 2 = 2.15$

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 2.15^2\right) \times 8.5 \\ &\approx 123.374\ 525 \end{aligned}$$

The volume of the cylindrical element is 123.374 525 m^3.

Enter the values in a table to see which element has the greatest volume.

Element	Length (m)	Diameter (m)	Volume (m_3)
Zarya FGB	12.6	4.1	166.267 71
Unity Node 1	5.5	4.6	91.3583
Zvezda service module	13.1	4.2	181.400 94
Z1 Truss	4.9	4.2	67.852 26
P6 Truss solar array	73.2	10.7	6578.824 38
Destiny	8.5	4.3	123.374 525

The element with the greatest volume is the P6 Truss solar array. Its volume is 6578.824 38 m^3.

b. Estimates may vary. Example: To estimate the total volume, use the following approximations: 170, 90, 180, 70, 6600, and 120.

$170 + 90 + 180 + 70 + 6600 + 120 = 7230$

An estimate for the total volume is 7230 m^3.

To calculate the total volume, add the last column of the table.

$$\begin{aligned}\text{total volume} &= 166.267\ 71 + 91.3583 + 181.400\ 94 + 67.852\ 26 + 6578.824\ 38 + 123.374\ 525 \\ &= 7209.078\ 115\end{aligned}$$

The total volume of the International Space Station is 7209.078 115 m^3.

12. To determine the better buy, find the volume of each container.

Jumbo container

The diameter of the container is 20 cm. The radius is half the diameter:

$r = 20 \div 2 = 10.$

volume of a cylinder = area of circular base × height of cylinder

$$\begin{aligned}V &= (\pi \times r^2) \times h \\ &\approx (3.14 \times 10^2) \times 40 \\ &\approx 12\ 560\end{aligned}$$

The volume of the Jumbo container is 12 560 cm^3.

Popcorn Lover's container

The diameter of the container is 30 cm. The radius is half the diameter:

$r = 30 \div 2 = 15$

volume of a cylinder = area of circular base × height of cylinder

$$\begin{aligned} V &= (\pi \times r^2) \times h \\ &\approx (3.14 \times 15^2) \times 20 \\ &\approx 14\,130 \end{aligned}$$

The volume of the Popcorn Lover's container is 14 130 cm^3.

The Popcorn Lover's container is a better buy because it has a larger volume.

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Going Beyond

1. $V = (\pi \times r^2) \times h$ is based on the 3-D object having a circular end. The term $(\pi \times r^2)$ represents the area of the circular base of a cylinder.

The Quonset huts in the pictures just have a semi-circular base, not a circular one. This means the structure has a base of only half the area of the base of a cylinder of the same radius. So $\left(\pi \times r^2\right)$ has to be replaced by $\frac{1}{2}\left(\pi \times r^2\right)$ in the formula.

Therefore, the formula for the volume of a Quonset hut is $V = \frac{1}{2}\left(\pi \times r^2\right) \times h.$

2. $$\begin{aligned} r &= \frac{d}{2} \\ &= \frac{6}{2} \\ &= 3 \end{aligned}$$

For a Quonset hut, $V = \frac{1}{2}\left(\pi \times r^2\right) \times h.$

$$\begin{aligned} V &= \frac{1}{2}\left(3.14 \times 3^2\right) \times 12 \\ &\approx \left(3.14 \times 9\right) \times 12 \\ &\approx \left(28.26\right) \times 12 \\ &\approx 14.13 \times 12 \\ &\approx 169.56 \end{aligned}$$

The volume of the Quonset hut in the illustration is 169.56 m^3.

Lesson 8

SC 1.

Car box volume	**Volume of crate**
$V = l \times w \times h$	$V = l \times w \times h$
$= 50 \times 30 \times 20$	$= 140 \times 120 \times 110$
$= 3000$	$= 1\,848\,000$
The volume of a car box is 3000 cm^3.	The volume of the crate is 1 848 000 cm^3.

$1\,848\,000 \div 30\,000 = 61.6$

Since only a whole number of car boxes will fit, a reasonable estimate is that 61 boxes will fit into the crate. Your estimate, however, may be fewer than this if you recognize that some space will be lost between some boxes.

SC 2. The original volume is 6.0288 m^3. Use this formula to determine the volume of the original cylinder.

$$V = \left(\pi \times r^2\right) \times h$$
$$\approx \left(3.14 \times 0.4^2\right) \times 12$$
$$\approx 6.0288$$

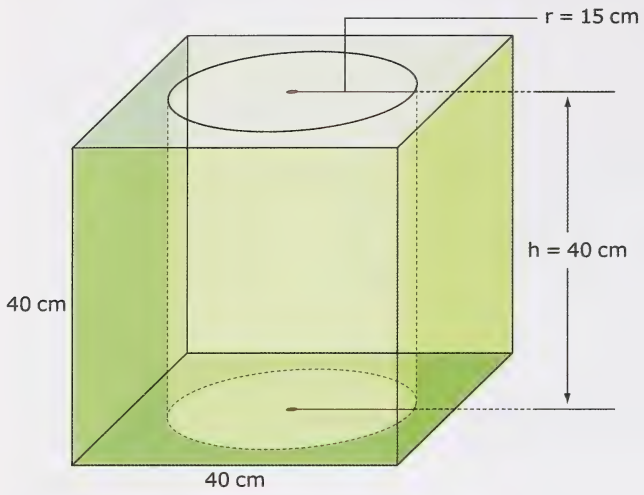
Then determine the new length (h) of the pipe.

$$6.0288 \approx (3.14 \times 0.6^2) \times h$$
$$\frac{6.0288}{1.1304} \approx (1.1304) \times \frac{h}{(1.1304)}$$
$$5.333333 \approx h$$

The length of the pipe must be 5.3 m (to the nearest tenth of a metre).

SC 3.

Visualizing a cylindrical tin can inside a cardboard box may help understand the diagram.
The following is a diagram of cube with cylindrical section removed.



Volume of the cube

$$V = s^3$$
$$= 403$$
$$= 64\ 000$$

The volume of the cube is 64 000 cm^3.

Volume of cylindrical space

$$V = (\pi \times r^2) \times h$$
$$\approx \left(3.14 \times 15^2\right) \times 40$$
$$\approx 28\ 260$$

The volume of cylindrical space is 28 260 cm^3.

Remaining volume of the cube

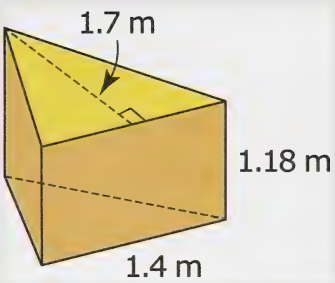
The remaining volume of the cube is 35 740 cm^3 (to the nearest whole cubic centimetre).

$$V \approx 64\ 000 - 28\ 260$$
$$\approx 35\ 740$$

The remaining volume of the cube is 35 740 cm^3 (to the nearest whole cubic centimetre).

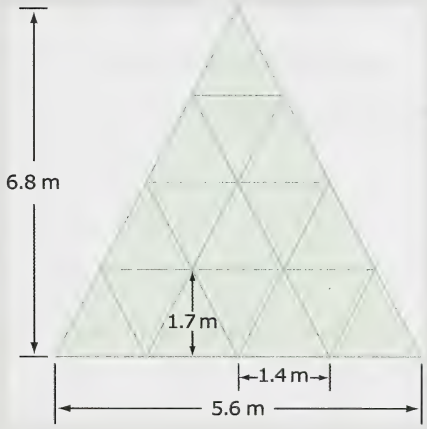
SC 4.

3.a. To build a giant prism with a triangular base of length 5.6 m and height 6.8 m, the artist requires four prisms on the bottom layer. Layer 2 would contain three prisms inverted in the cavities. Layer 3 would contain three prisms. Layer 4 would contain two prisms inverted in the cavities. Layer 5 would contain two prisms. Layer 6 would contain one prism inverted in the cavity. Layer 7 would contain one prism. See diagram below.



total length of base = 4 × 1.4 = 5.6
total height of prism = 4 × 1.7 = 6.8

This model contains 4 + 3 + 3 + 2 + 2 + 1 + 1 = 16 prisms. The artist has enough prisms to build the giant triangular prism.



b. To determine the volume of the giant prism, find the volume of one of the small prisms.

$$\text{volume of a triangular prism} = (\text{base} \times \text{height} \div 2) \times \text{height of prism}$$

$$V = (1.4 \times 1.7 \div 2) \times 1.18$$

$$V = 1.4042$$

The volume of one triangular prism is 1.4042 m^3.

To find the volume of the giant prism, multiply 1.4042 m^3 by 16:

$V = 1.40 \times 16$
$V = 22.47$

The volume of the giant prism is 22.47 m^3, to the nearest hundredth of a cubic metre.

4. Determine the volume of the first cylinder.

The diameter of the cylinder base is 10 cm. The radius is half the diameter: $r = 10 \div 2 = 5$.

volume of a cylinder = area of circular base × height of cylinder

$$V = \left(\pi \times r^2\right) \times h$$
$$V \approx \left(3.14 \times 5^2\right) \times 30$$
$$V \approx 2355$$

The volume of the cylinder is 2355 cm^3.

To find the height of the second cylinder, divide the volume by the area of the base.

The diameter of the cylinder base is 8 cm. The radius is half the diameter: $r = 8 \div 2 = 4$.

the area of the base $= (\pi \times r^2)$

$$A \approx 3.14 \times 4^2$$
$$A \approx 50.24$$

The area of the base is 50.24 cm^2.

height of cylinder = volume of cylinder ÷ area of circular base

$$h = 2355 \div 50.24$$
$$h = 46.875$$

The height of the second cylinder to the nearest tenth of a centimetre is 46.9 cm.

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SC 5.

8. To determine whether the volume of one crate will be enough, find the volume of the crate in cubic centimetres.

Convert the measurements to centimetres:

3 m = 300 cm; 7 m = 700 cm

Find the volume of the crate:

volume of a rectangular prism = (length × width) × height

$V = l \times w \times h$
$V = 300 \times 300 \times 700$
$V = 63\ 000\ 000$

The volume of the crate is 63 000 000 cm^3.

Find the volume of each box.

volume of a rectangular prism = (length × width) × height

$V = l \times w \times h$
$V = 10 \times 10 \times 20$
$V = 2000$

The volume of each box is 2000 cm^3.

To find the volume of 25 000 boxes, multiply the number of boxes by the volume:

$V = 25\ 000 \times 2000$
$V = 50\ 000\ 000$

The volume of 25 000 boxes is 50 000 000 cm^3. The crate is large enough to hold the boxes.

Check that you could stack them. Place the 10 cm × 10 cm bases of the boxes directly on the 300 cm × 300 cm base of the box. This layer will have 30 boxes along each two perpendicular sides of the base of the box. So this layer will have 900 boxes and be 20-cm high.

$700 \div 20 = 35$

You could make a total of 35 layers of boxes in the crate this way.

$35 \times 900 = 31\ 500$

These layers would hold 31 500 boxes.

Therefore, one crate can hold the 25 000 boxes to be shipped.

14. To determine the price per cubic centimetre, first find the volume of the small jar.

The diameter of the cylinder base is 8 cm. The radius is half the diameter: $r = 8 \div 2 = 4$.

volume of a cylinder = area of circular base × height of cylinder

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 4^2\right) \times 10 \\ &\approx 502.4 \end{aligned}$$

The volume of the small jar is 502.4 cm^3.

To determine the price of peanut butter per cubic centimetre, divide \$1.60 by 502.4.

\$1.60 ÷ 502.4 cm^3 = \$0.003 18/cm3 or 0.318¢/cm^3

Find the volume of the large jar.

The diameter of the cylinder base is 16 cm. The radius is half the diameter: $r = 16 \div 2 = 8$.

volume of a cylinder = area of circular base × height of cylinder

$$\begin{aligned} V &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 8^2\right) \times 10 \\ &\approx 4019.2 \end{aligned}$$

The volume of the large jar is 4019.2 cm^3.

To determine the price of the large jar, multiply the volume of the large jar by the price per cubic centimetre of the small jar:

$4019.2 \times 0.318 = 1278.1$¢ or \$12.78

The price of the large jar should be \$12.78.

17. Answers may vary. Example: One way of rolling the paper is to use 22 cm as the height and 28 cm as the circumference of the circular base.

Find the radius of a circle with circumference 28:

$$\text{radius} = \text{circumference} \div (2 \times \pi)$$
$$r = 28 \div (2 \times 3.14)$$
$$r = 4.4586$$

The radius of the cylinder is 4.4586 cm.

$$\text{volume of a cylinder} = (\pi \times r^2)h$$
$$V = (3.14 \times 4.45862) \times 22$$
$$= 1373$$

The volume of a cylinder with height of 22 cm and circumference of 28 cm is 1373 cm^3.

The second way of rolling the paper is to use 28 cm as the height and 22 cm as the circumference of the circular base.

Find the radius of a circle with circumference 22:

$$\text{radius} = \text{circumference} \div (2 \times \pi)$$
$$r = 22 \div (2 \times 3.14)$$
$$r = 3.5$$

The radius of the cylinder is 3.5 cm.

$$\text{volume of cylinder} = (\pi \times r^2) \times h$$
$$V = (3.14 \times 3.5^2) \times 28$$
$$V = 1077.0$$

The volume of a cylinder with a height of 28 cm and circumference of 22 cm is 1077.0 cm^3. The cylinder with a height of 22 cm and circumference of 28 cm forms the largest volume.

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Going Beyond

21.
a. Find the volume of the rectangular-prism portion of the tunnel. Change the measurement of 4 km to metres: 4 km = 4000 m. Find the height of the rectangular prism by subtracting half the diameter from 8 m:

$8 - 2.5 = 5.5$

The height of the rectangular prism is 5.5 m.

volume of a rectangular prism = $l \times w \times h$
$V = 5 \times 4000 \times 5.5$
$V = 110\,000$

The volume of the air in the rectangular-prism portion of the tunnel is 110 000 m^3.

Find the volume of the half-cylinder portion of the tunnel. To do this, find the volume of a cylinder with diameter 5 cm and height 4000 m and divide by 2.

The diameter of the tunnel is 5 m. The radius is half the diameter: $r = 5 \div 2 = 2.5$.

$$\begin{aligned} \text{volume of a cylinder} &= \left(\pi \times r^2\right) \times h \\ &\approx \left(3.14 \times 2.5^2\right) \times 4000 \\ &\approx 78\,500 \\ 78\,500 \div 2 &= 39\,250 \end{aligned}$$

The volume of the half-cylinder portion of the tunnel is 39 250 m^3.

The total volume of air in the tunnel is 110 000 + 39 250 = 149 250 m^3.

b. To determine how many hours and minutes it would take to replace the stale air, first find the answer in seconds. To do this, divide the volume of the air by 10 m^3 per second:

$149\,250 \div 10 = 14\,925$

Change 14 925 seconds to minutes by dividing by 60: $14\,925 \div 60 = 248.75$

Change 248.75 minutes to hours by dividing by 60: $248.75 \div 60 = 4.15$

It would take 4.15 hours, or 4 hours and 9 minutes, to replace the stale air.

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Unit 4 Summary

SC 1. Turn to pages 491 and 492 of the Answers section of your textbook. Compare your answers with those provided in the textbook.

SC 2. Turn to page 497 of the Answers section of your textbook. Check your answers with those provided.

SC 3.

1. D

The top view of this container shows a rectangle.

3. D

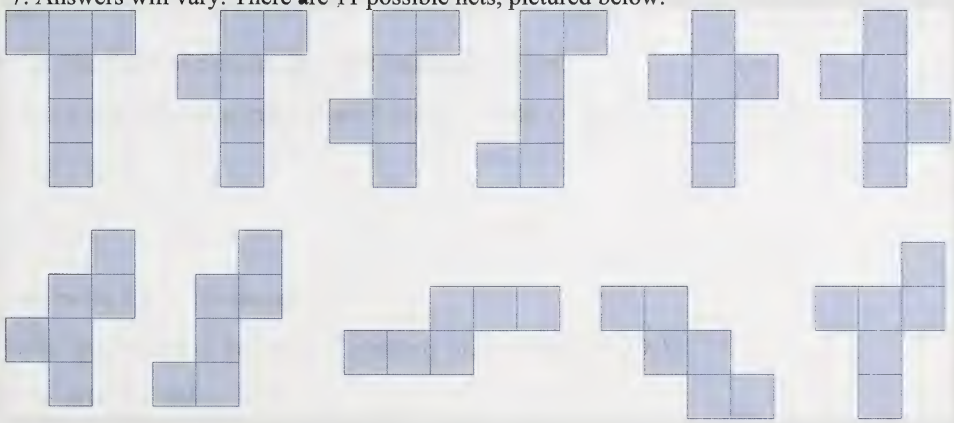
A rectangular prism has a net like the one shown.

5. C

The surface area of the cylinder is 631.14 cm^2.

$$\begin{aligned} SA &\approx 2\times 3.14\times 3^2 + 3.14\times 6\times 30.5 \\ &\approx 56.52 + 574.62 \\ &\approx 631.14 \end{aligned}$$

7. Answers will vary. There are 11 possible nets, pictured below.



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9. Find the area of one of the congruent faces.

area of one square face: $1014 \div 6 = 169$

Find the side length of one square face:

$\sqrt{169} = 13$

The length of a side of the cube is 13 cm.

11. Find the surface area of each cylinder and compare.

Ken's cylinder

$$\begin{aligned} SA &\approx 2 \times 3.14 \times 4.5^2 + 3.14 \times 9 \times 18 \\ &\approx 127.17 + 508.68 \\ &\approx 635.85 \end{aligned}$$

The surface area of Ken's cylinder is 635.85 cm^2.

Arika's cylinder

$$\begin{aligned} SA &\approx 2 \times 3.14 \times 2.25^2 + 3.14 \times 4.5 \times 36 \\ &\approx 31.7925 + 508.68 \\ &\approx 540.4725 \end{aligned}$$

The surface area of Arika's cylinder is 540.47 cm^2.

Ken's cylinder has a larger surface area.

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SC 4.

1. C

volume of a right rectangular prism = (length × width) × height

$$\begin{aligned} V &= l \times w \times h \\ V &= 12 \times 5.5 \times 2 \\ V &= 132 \end{aligned}$$

The volume of the right rectangular prism is 132 cm^3.

3. D

volume of a cube = (area of base) × height

$V = s \times s \times s$
$V = 8 \times 8 \times 8$
$V = 512$

The volume of the cube is 512 cm^3.

5. B

Find the volume of the right rectangular prism.

volume of a right rectangular prism = (length × width) × height

$V = l \times w \times h$
$V = 30 \times 25 \times 12$
$V = 9000$

The volume of the trough is 9000 cm^3.

Find the volume of the cylindrical heater.

volume of a cylinder = area of circular base × height of cylinder

$V = \left(\pi \times r^2\right) \times h$
$V \approx \left(3.14 \times 5^2\right) \times 12$
$V \approx 942$

The volume of the cylinder is 942 cm^3.

To find the maximum volume of water in the trough in winter, subtract the volume of the heater from the volume of the trough:

9000 – 942 = 8058

The maximum volume of water in the trough in winter is 8058 cm^3.

7. volume of a right rectangular prism = (length × width) × height

$V = l \times w \times h$
$V = 3 \times 4 \times 6$
$V = 72$

The volume of the right rectangular prism is 72 cm^3.

9. Find the volume of one of the rectangular prism-shaped boxes.

volume of a right rectangular prism = (length × width) × height

$V = l \times w \times h$
$V = 29.5 \times 18 \times 9.5$
$V = 5044.5$

The volume of one of the boxes is 5044.5 cm^3.

To find the volume of 12 boxes, multiply 12 by the volume of one box:

12 × 5044.5 = 60 534

Twelve of these boxes will take up 60 534 cm^3 of Ying's closet.

11. Find the volume of the cylindrical storage container.

volume of a cylinder = area of circular base × height of cylinder

$V = \left(\pi \times r^2\right) \times h$
$V \approx \left(3.14 \times 7^2\right) \times 80$
$V \approx 12\,308.8$

The volume of the cylindrical storage container is 12 308.8 cm^3.

Find the volume of the right triangular prism-shaped storage container.

volume of a right triangular prism = (base × height ÷ 2) × height of prism

$V = \left(20 \times 14 \div 2\right) \times 80$
$V = 11\,200$

The volume of the triangular prism-shaped container is 11 200 cm^3.

The cylindrical storage container is larger because it has a larger volume.

13. Find the volume of the rectangular prism-shaped bin.

volume of a right rectangular prism = (length × width) × height

$V = l \times w \times h$
$V = 2.5 \times 2 \times 2$
$V = 10$

The volume of garbage bin is 10 m^3.

Find the volume of a cylindrical garbage can. The diameter of the cylinder base is 0.75 cm. The radius is half the diameter:

$r = 0.75 \div 2 = 0.375$

volume of a cylinder = area of circular base × height of cylinder

$V = \left(\pi \times r^2\right) \times h$
$V \approx \left(3.14 \times 0.375^2\right) \times 1.2$
$V \approx 0.53$

The volume of a cylindrical garbage can is 0.53 m^3.

To determine how many garbage cans will fill the bin, divide the volume of the bin by the volume of a garbage can:

$10 \div 0.53 = 18.87$

To the nearest full can, 19 cylindrical garbage cans can be emptied into the bin before it is full.

15.a. The shape of the patio is a right rectangular prism.

volume of a right rectangular prism = (length × width) × height

$V = l \times w \times h$
$V = 6 \times 6 \times 0.15$
$V = 5.4$

Yuri needs 5.4 m^3 of concrete.

b. To find the cost of the concrete, multiply the volume by $110.00:

$5.4 \times 110.00 = 594$

Yuri has to pay $594.00 before tax.

MathLinks 8: Solutions CD (Toronto: McGraw-Hill Ryerson, 2008) 325-331. Reproduced by permission.

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